考試科目	數理統計學	系所別	統計學系	考試時間	2 月 9 日(三) 第二節

1. (23%) Two machines of the same type, A and B, always operate under the same environment, but an extra filter had installed on A to reduce the pollutant. Let Y_1 be the amount of pollutant emitted by A, while Y_2 be the pollutant amount emitted by B. The joint pdf is

$$f(y_1,y_2)=\begin{cases}1/2, & 0\le y_1\le 1, 0\le y_2\le 4, 4y_1\le y_2,\\ 0, & \text{otherwise.}\end{cases}$$

(1) Find the probability that Y_2/Y_1 is at least 5. (5%)
(2) Find the condition pdf of Y_1 given $Y_2=y_2$, and $V(Y_1\mid y_2)$. (8%)
(3) The reduction of pollutant amount due to the filter is $R=Y_2-Y_1$. Find the probability density function of R and its mean. (10%)

2. (22%) Let $X_1,\cdots,X_n$ be a random sample from $f(x;\theta)=\frac{1}{\theta}$, $|x|\le\frac{\theta}{2}$; $\theta>0$.

(1) Find the complete and sufficient statistic for θ. (10%)
(2) Find the MVUE (minimum variance unbiased estimator) of $\theta^{-1}+\theta$. (12%)

3. (15%) Given that $N=n$, the conditional pdf of X is $f(x\mid n)=\frac{x^{n-1}e^{-x}}{\Gamma(n)}$, $x>0$.

Moreover, $P(N=n)=\frac{\mu^n e^{-\mu}}{n!}$, $n=0,1,2,\ldots$.

(1) Find the unconditional mean and variance of X, i.e. $E(X)$ and $V(X)$. (5%)
(2) Find the limiting distribution for $\frac{X-E(X)}{\sqrt{V(X)}}$ as $\mu\to\infty$. (10%)

4. (40%) Let $X_1,\cdots,X_n$ be a random sample from $f(x;\theta)=\lambda x^{-(\lambda+1)}$, $x>1$; $\lambda>0$.

(1) Find the MLE (maximum likelihood estimator) of λ. (5%)
(2) Find the Cramer-Rao lower bound for the variance of unbiased estimators of $P(X>a)$, where a is a constant greater than 1. (7%)
(3) Find the UMP size α test of $H_0:\lambda\le\lambda_0$ versus $H_1:\lambda>\lambda_0$. (10%)
(4) Find the 100(1-α)% confidence interval for λ. (6%)
(5) Let $Y_1,\cdots,Y_m$ be another random sample from $f(y;\theta)=\theta y^{-(\theta+1)}$, $y>1$; $\theta>0$. Derive the likelihood ratio test of $H_0:\lambda=\theta$ versus $H_1:\lambda\ne\theta$. (12%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。