考 試 科 目	基礎數學	系 所 別	統計學系	考 試 時 間	2月 9日 (三) 第一節

1. (36pts). For each of the following statements, determine whether it is true (◯) or false (╳). (True or false questions. Do not give explanation.)

(a) $\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}$ does not exist.

(b) If functions $f(x)$ and $g(x)$ are continuous on $[a,b]$, then the following definite integrals exist and $\int_a^b f(x)g(x)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x \int_a^b g(x)\mathrm{d}x$.

(c) The equations (i) $x^2+y^2=9$, (ii) $x=-3\cos 2t, y=-3\sin 2t$ $(0\le t\le \pi)$, and (iii) $r=3$ all have the same graph.

(d) Suppose that the radius of convergence of the power series $\Sigma_{k=0}^{\infty} a_k x^k$ is R_0 for $0<R_0<\infty$. Then the radius of convergence of the power series $\Sigma_{k=0}^{\infty} a_k x^{2k}$ is R_0^2.

(e) The function $f(x,y)=\begin{cases}\frac{x^2y}{x^2+y^4} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ is continuous at $(0,0)$.

(f) Suppose $f(x,y):\mathcal{R}\times\mathcal{R}\longrightarrow\mathcal{R}$ has continuous second partial derivatives on $(-\delta,\delta)\times(-\delta,\delta)$ for some $\delta>0$, and suppose that $f_x(0,0)=0$ and $f_y(0,0)=0$. If the matrix $\begin{bmatrix} f_{xx}(0,0) & f_{xy}(0,0)\\ f_{yx}(0,0) & f_{yy}(0,0)\end{bmatrix}$ is positively definite, then f has a local minimum at $(0,0)$.

(g) A homogeneous system of four equations in six unknowns always has nontrivial solutions.

(h) If A is nonsingular and $A^{-1}=A^T$, then $\det(A)=\pm 1$.

(i) If W_1 and W_2 are subspaces of a vector space V, then $W_1\bigcup W_2$ is also a subspace of V.

(j) Let A and B be $m\times n$ and $n\times m$ matrices respectively. Then AB and BA are well defined and rank(AB) =rank(BA).

(k) Let A be 3×3 matrix. If A has only two distinct eigenvalues λ_1 and λ_2, then A is not diagonalizable.

(l) Suppose $A_{n\times n}$ is similar to a diagonal matrix $D_{n\times n}=[d_{ij}]_{1\le i,j\le n}$. Namely, $A=PDP^{-1}$ for some nonsingular matrix $P_{n\times n}$. Then $\Sigma_{k=1}^{\infty}A^k$ exists if $|d_{ii}|<1$ for all $i=1,2,\ldots,n$.

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2. (12pts). Please sketch the graph of the polynomial $f(x) = 4x^5 + x^3 - 2x + 1$ and find the integer a such that the only real root of $4x^5 + x^3 - 2x + 1 = 0$ is in $(a, a+1)$. Show your work.

3. (10pts). Please use appropriate power series to show $\ln 2 = \Sigma_{k=1}^{\infty} \frac{1}{k2^k}$.

4. (10pts). Please evaluate $\int\int_D \frac{(x+2y)\sqrt{x-y+3}}{x+2y+1}\, dA$ by making an appropriate change of variables, where D is the parallelogram enclosed by the lines $x + 2y = 0, x + 2y = 4, x - y = 1$, and $x - y = 6$. Show your work.

5. (12pts). Let W be the subspace of $\mathcal{R}^4$ spanned by the vectors $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, and $\mathbf{w}_3 = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$. Please find the (orthogonal) projection of $\mathbf{v} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ on W. Show your work.

6. (10pts). Consider the vector space P_2 of all polynomials of degree ≤ 2 together with the zero polynomial. Note $S = \{p_1(t) = t^2 + t + 1, p_2(t) = t + 1, p_3(t) = 1\}$ is a basis for P_2. Let $L : P_2 \longrightarrow P_2$ be the linear transformation whose matrix with respect to S is $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$. Please find $L(at^2 + bt + c)$. Show your work.

7. (10pts). Records about the migration patterns of a species of bird from year to year among three habitats A, B and C show that of the birds beginning of the year in habitat A, 20% migrate to habitat B, and 50% migrate to habitat C. Of the birds beginning of the year in habitat B, 10% migrate to habitat A, and 10% migrate to habitat C. Of the birds beginning of the year in habitat C, 40% migrate to habitat A, and 40% migrate to habitat B. In the long wrong, what percentage of the birds will live in each of the habitats? Show your work.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。