考試科目	微積分	系所別	國貿所	考試時間	2月10日(四)第4節

※ Show all your work for full credit.

1. Evaluate the following integrals. (20%)

   (a) $\int_1^8 \frac{1}{x+2\sqrt[3]{x}}dx$.

   (b) $\int_0^9 |\sqrt{x}-1|dx$

   (c) $\int_0^1 x3^x dx$.

   (d) $\int_0^1 x^3\sqrt{1-x^2}dx$

2. Let f be a differentiable function such that $x^3f(x)+f(x^2)=4$ for all $x>0$. Find $f'(1)$. (10%)

3. Let a and b be real numbers such that $\lim_{x\to\infty}\left(xe^{1/x}-(ax+b)\right)=0$. Find the values of a and b. (10%)

4. Find $\frac{dy}{dx}$ at $x=1$ if $x^y+xy=4$. (10%)

5. Find the maximum and minimum values of the function $f(x,y)=x+2y-z$ subject to the constraint $x^2+y^2+z^2=6$. (10%)

6. Determine the interval of convergence for the series: $\sum_{n=1}^{\infty}\frac{(-1)^n(2x-1)^n}{n3^n}$. (10%)

7. Evaluate the double integral $\iint_R 8x^3y\,dxdy$, where R is the rectangle with vertices $(-1,0)$, $(2,0)$, $(2,3)$, $(-1,3)$. (10%)

8. Find $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$. (10%)

9. Let $f(x)=(\ln x)^x$. Find $f'(x)$. (10%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。