考 試 科 目	統計學	系 所 別	國際經營與貿易學系	考 試 時 間	2月10日（星期四）第四節

I. Multiple Choice (4 points each)

Identify the letter of the choice that best answers the question.

> Upper-tail critical values of chi-square distribution with different degrees of freedom:
>
> $\chi^2_{1,5\%} = 3.841$; $\chi^2_{2,5\%} = 5.991$; $\chi^2_{3,5\%} = 7.815$; $\chi^2_{4,5\%} = 9.488$; $\chi^2_{5,5\%} = 11.070$; $\chi^2_{6,5\%} = 12.592$.
>
> Critical points of a stand normal distribution: $Z_{0.95} = 1.645$; $Z_{0.975} = 1.96$.

1. The variables x (temperature in °C) and y (temperature in °F) are related by the formula y = 32 + 1.8x. Therefore, the correlation between x and y will be:

   A. 1.8, because if x increases by 1°C then y increases by 1.8°F.

   B. 0, because if x = 0 then y = 32.

   C. −1, because if x decreases then y decreases.

   D. 1, because the variables have a deterministic, linear, increasing relationship.

2. Suppose you are playing a game and you have 0.2 chance of winning. Suppose you decide to play repeatedly until you win. The games are independent. What is the probability that you win on the first or second try?

   A. 0.2 × 0.8 = 0.16.

   B. 0.2 × 0.2 = 0.04.

   C. 0.2 + (0.2 × 0.8) = 0.36.

   D. 0.2.

3. Suppose the probability that a child lives with his or her mother as the sole parent is 0.258, and the probability that a child lives with his or her father as sole parent is 0.037. Then the probability that a child either lives with both or with neither parent is:

   A. 0.258 + 0.037 = 0.295.

   B. 1 − 0.037 = 0.963.

   C. 1 − 0.258 = 0.742.

   D. 1 − (0.258 + 0.037) = 0.705.

4. If you have a data set that consists of the following three values 1, 2, and 3, which of the following statements are true:

   A. The range of the data is 3.

   B. The sample standard deviation equals the sample average.

   C. The sample standard deviation equals the sample variance.

   D. None of the above are true.

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5. Assume the model: $Y = 5 + 2X_1 + 3X_2 + \varepsilon$, $\varepsilon \sim N(0, 81)$. What is the $Var[Y \mid X_1 = 0, X_2 = 4]$?

    A. 9.

    B. 81.

    C. 17.

    D. 6.

Question 6 and 7.

6. Suppose that 1% of the population are infected with the COVID-19 coronavirus. Suppose we have a test for the infection that has 80% sensitivity (the ability of a test to correctly identify patients with an infection) and 90% specificity (the ability of a test to correctly identify people without the infection). What is the probability of a false positive?

    A. 0.99.

    B. 0.10.

    C. 0.01.

    D. 0.20.

7. Following the above question, what is Pr(COVID-19-infected given that the test is positive)?

    A. 0.80.

    B. 0.14.

    C. 0.01.

    D. 0.075.

8. In a random sample of 1000 college graduates, 75% stated that they prefer bubble tea (over coffee). Therefore, a 90% confidence interval for the proportion of all college graduates who prefer bubble tea is:

    A. $0.75 \pm 1.645 \times \sqrt{0.75/1000}$.

    B. $0.75 \pm 1.96 \times \sqrt{0.75 \times 0.25/1000}$.

    C. $0.75 \pm 1.645 \times \sqrt{0.75 \times 0.25/1000}$.

    D. $0.75 \pm 1.96 \times \sqrt{0.75/1000}$.

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9. A statistical study considers the question of whether highly educated people are less likely to develop dementia than others. In this study, the alternative hypothesis is:

    A.   Insufficient information is given to allow us to determine the null hypothesis.

    B.   There is a relationship between level of education and the development of dementia.

    C.   There is no relationship between level of education and the development of dementia.

    D.   Highly educated people are less likely than others to develop dementia.

10. A study of undergraduate students failed (p-value = 0.22) to find a difference in pulse rates between men and women. Which of the following is true?

    A.   The null hypothesis is rejected.

    B.   It is possible that they committed a Type 1 error.

    C.   The research hypothesis is supported.

    D.   It is possible that they committed a Type 2 error.

II. Problem-Solving Questions:
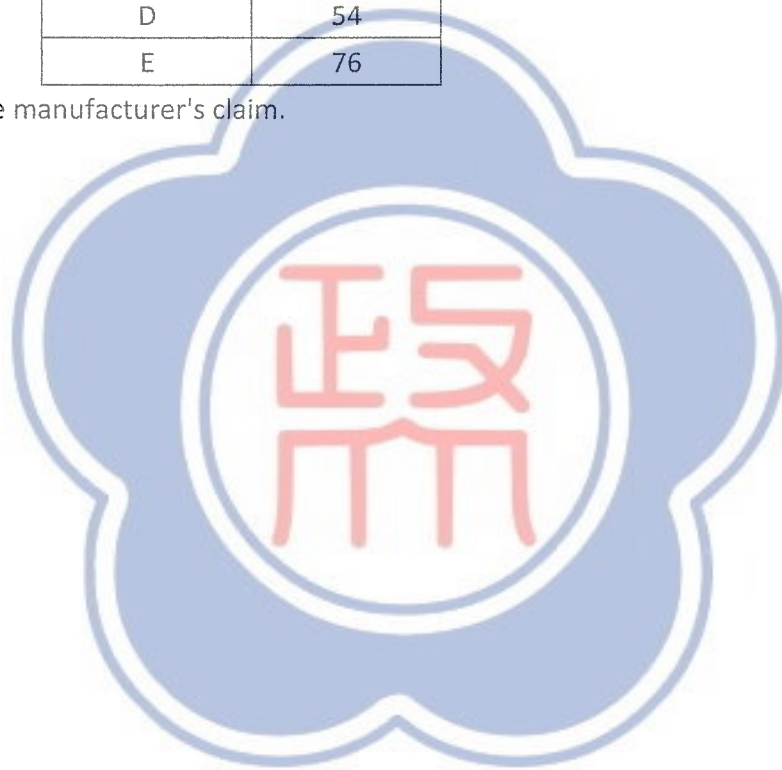
Please answer the following questions IN SEQUENCE.

1. 30 people took a COVID-19 PCR test. However, the doctor was so careless that she/he forgot to label the test. She/he had to give back all the test results randomly, with one unique sample to each person. In particular, each person gets her/his own test result with probability 1/30. Let X be the number of people who get their own test results.

    A.   (10 points) What is the expected value of X?

    B.   (10 points) What is the variance of X?

2. (20 points) In manufacturing its smartphone, SmartComm buys a particular kind of microchip from 3 suppliers: 30% from Samsung, 20% from MediaTek, and 50% from TSMC. SmartComm has extensive histories on the reliability of the chips and knows that 5% of the chips from Samsung are defective; 4% from MediaTek are defective and 2.5% from TSMC are defective. In testing a newly assembled smartphone, SmartComm found the microchip to be defective. Which provider is the likely culprit?

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3. (20 points) A major electric-vehicle (EV) manufacturer claimed that the frequencies of repairs on all five models of its EV are the same. A sample of 300 repair services showed the following frequencies on the various makes of EV.

Model	Frequency
A	42
B	65
C	63
D	54
E	76

At $\alpha = 0.05$, test the manufacturer's claim.

備　　　註	一、作答於試題上者,不予計分。 二、試題請隨卷繳交。