

考 試 科 目	統 計 學	系 所 別	經 濟 學 系	考 試 時 間	2 月 9 日(三)第四節
---------	-------	-------	---------	---------	---------------

- 共 7 大題，合計 100 分。
- 請依題號順序作答。
- 答題若無詳述理由或計算推導過程，不予計分。

1. (Total 15%) The probability density function of X is given by

$$f(x) = kx^2, \quad 0 < x < 3$$

where k is a constant.

- (5%) Find k .
- (5%) Find the expected value of X .
- (5%) Find the cumulative distribution function of X .

2. (Total 15%) The joint distribution of X and Y is given below.

	$Y = 1$	$Y = 2$
$X = 1$	0.10	0.20
$X = 2$	0.60	0.10

- (5%) Find $E[Y | X = 1]$.
 - (5%) Find the probability distribution of $E[Y | X]$.
 - (5%) Find $E[E[Y | X]]$.
3. (Total 14%) Let \bar{X} denote the sample average from a random sample with mean μ and variance σ^2 . Consider an estimator $W = \frac{n-1}{n}\bar{X}$ of μ , where n is the sample size.
- (3%) Show that W is a biased estimator of μ and find the bias.
 - (3%) What happens to the bias of W as $n \rightarrow \infty$?
 - (3%) Find the variance of W .
 - (5%) Is W a consistent estimator of μ ? Explain.

考試科目	統計學	系所別	經濟學系	考試時間	2月9日(三)第四節
------	-----	-----	------	------	------------

- 共 7 大題，合計 100 分。
- 請依題號順序作答。
- 答題若無詳述理由或計算推導過程，不予計分。

4. (Total 10%) Show that for random variables Y and X , if $E[Y | X] = E[Y]$ then $Cov(X, Y) = 0$.

5. (Total 8%) Suppose that the logarithm of wages ($\log(wage)$) is related to years of schooling ($educ$) and work experience ($exper$, in years) by the conditional expectation:

$$E[\log(wage) | educ, exper] = \beta_0 + \beta_1 educ + \beta_2 exper$$

where $\beta_0 = 1.05$, $\beta_1 = 0.10$, and $\beta_2 = 0.08$.

(a) (3%) Interpret β_1 .

(b) (5%) Suppose that the expected values of $educ$ and $exper$ are 12 years and 20 years, respectively. Find $E[\log(wage)]$. Show your arguments.

6. (Total 15%) Consider the following demand and supply equations:

$$\text{Demand: } Q = -\beta_1 P + e_1$$

$$\text{Supply: } Q = \beta_2 P + e_2$$

where Q and P denote quantities and prices, respectively, and the demand error e_1 and supply error e_2 satisfy $E[e_1] = E[e_2] = 0$, $Var(e_1) = Var(e_2) = 1$, and $Cov(e_1, e_2) = 0$.

(a) (5%) Find the equilibrium price and quantity.

(b) (5%) Compute $Cov(P, e_1)$ in terms of β_1 and β_2 .

(Hint: Use your answers in (a).)

(c) (5%) Given a random sample $\{P_i, Q_i\}_{i=1}^n$ of equilibrium prices and quantities, consider the following linear regression:

$$Q_i = \gamma_0 + \gamma_1 P_i + u_i$$

where $E[u_i] = 0$ and $E[P_i u_i] = 0$. Find γ_1 in terms of β_1 and β_2 .

(Hint: Use your answers in (a).)

考試科目	統計學	系所別	經濟學系	考試時間	2月9日(三)第四節
------	-----	-----	------	------	------------

- 共 7 大題, 合計 100 分。
- 請依題號順序作答。
- 答題若無詳述理由或計算推導過程, 不予計分。

7. (Total 23%) Consider a linear regression for a Cobb-Douglas production function in a log-linear form:

$$\log(Y) = \beta_0 + \beta_1 \log(L) + \beta_2 \log(K) + u$$

where Y is output, L is labor input, K is capital input, u is the error term, and $(\beta_0, \beta_1, \beta_2)$ are coefficients to be estimated.

Given a random sample $\{Y_i, L_i, K_i\}_{i=1}^n$ of $n = 525$ firms, the OLS estimates $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ are given below:

$$\widehat{\log(Y_i)} = 1.220 + 0.954 \log(L_i) + 0.256 \log(K_i)$$

and the covariance matrix estimate is:

$$\widehat{Var} \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 2 \\ 3 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix} \times 10^{-4}$$

- (3%) Interpret $\hat{\beta}_1$.
- (5%) Find the standard error of $\hat{\beta}_1 + \hat{\beta}_2$.
- (5%) You are interested in testing *constant returns to scale*. Write the associated testing hypotheses and the t test statistic.
- (5%) Based on the information above, calculate the t test statistic.
- (5%) Now suppose that *unobserved firm-specific productivity* positively affects firms' output as well as labor input. Which true value of β_1 , 0.724 or 1.132, do you think would be more likely? Explain your answer.

備

註

- 一、作答於試題上者, 不予計分。
- 二、試題請隨卷繳交。