【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (15%) An infinitely long, thin conducting circular cylinder of radius $b$ is split in four quarter-cylinders, as shown in Fig. 1. The quarter-cylinders in the second and fourth quadrants are grounded, and the first and third quadrants are kept at potential $V_0$ and $-V_0$, respectively. Determine the potential distribution both inside and outside the cylinder.

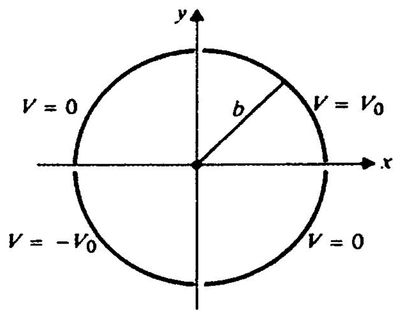


Fig. 1.

2. (15%) Legendre polynomial,

a). if the generating function of Legendre polynomial is, $g(x,t) = \frac{1}{\sqrt{1-2xt+t^2}} \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} P_n(x)t^n$ , show that the electrical potential for a point charge located at z-axis at $(0,0,a)$ can be written as (10%) $V(r,\theta,\phi) = \frac{q}{4\pi\varepsilon_0 r}\sum_{n=0}^{\infty} P_n(\cos\theta)\left(\frac{a}{r}\right)^n$

b). For r >>d, please determine the electrical potential of three charges $(+q,-2q,+q)$ are arranged along the z-axis at $z=\frac{d}{2}, z=0$ and $z=-\frac{d}{2}$, respectively. (you may need to use the Legendre polynomial $\begin{cases} P_0(x) = 1 \\ P_1(x) = x \\ P_2(x) = \frac{1}{2}(3x^2-1) \\ P_3(x) = \frac{1}{2}(5x^3-3x) \end{cases}$ (5%)

3. (20%) the vector potential $\mathbf{A}$ and scalar potential $\phi$ are not unique in that it is possible to add to $\mathbf{A}$ the gradient of a scalar field $\psi$, such that $\mathbf{A} = \mathbf{A} + \nabla\psi$

a). Show that the magnetic field $\mathbf{B}$ is unchanged (5%)

b). In order not to change the e-field $\mathbf{E}$, please find out the relation between $\phi$ and $\phi'$ (5%)

c). Discuss the condition that $\psi$ must satisfy so that the new potential $\mathbf{A}'$ and $\phi'$ remain governed by the non-homogeneous wave equation for vector and scalar potential (10%)

4. (32%)

(a) The velocity of light propagation at free space $\left(c = 3\times10^8\frac{m}{s}\right)$ depends on two physical constants, please state the name of these two constants, their values and the equation relating these two constants and $c$. (8%)

(b) Please define transverse electromagnetic (TEM) wave. (6%)

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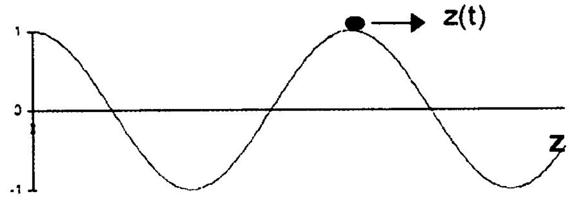
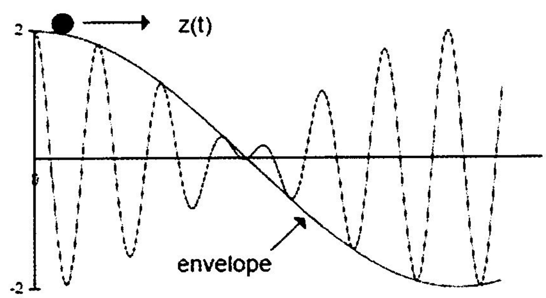
(c) The electric field intensity of a linearly polarized uniform plane wave propagating in the $+z$-direction is $E = 10\cos(10^7\pi t)\left(\frac{V}{m}\right)$. Phase velocity $u_p = \frac{\omega}{\beta}\left(\frac{m}{s}\right)$ describes the motion of a point, line or surface of constant phase for waves in one, two and three dimensions respectively, as shown in the figure below, where $\omega$ is the angular frequency and $\beta$ is the phase constant or propagation constant. Given the $\beta = 8.89\left(\frac{\text{rad}}{m}\right)$, please find the phase velocity and the wavelength of the wave. (8%)




(d) Given that a wave packet consists of two traveling waves having equal amplitude and slightly different angular frequency $\omega_0 + \Delta\omega$ and $\omega_0 - \Delta\omega$, where $\Delta\omega \ll \omega_0$, as shown in the figure below. The phase constants, being functions of frequency, will also be slightly different. Assume the phase constants corresponding to the frequencies be $\beta_0 + \Delta\beta$ and $\beta_0 - \Delta\beta$. Derive the group velocity equation. (4%)




(e) EM wave has many applications in daily life; please state the daily life applications of the EM wave at (i) wavelength of nm; (ii) wavelength of μm; (iii) wavelength of mm. (6%)

5. (18%) In a lossless $50\Omega$ transmission line terminated by an unknown loading, the standing-wave ratio is 3.0. The distance between successive voltage minima is 20 cm, and the first minimum is at 5 cm from the loading.
   (a) Find the reflection coefficient $\Gamma$ (8%)
   (b) Find the loading impedance $Z_L$ (8%)
   (c) Find the loading impedance if there is a maximum power transfer to the loading (2%)