國立政治大學 111 學年度 碩士班招生考試試題

考試科目	線性代數	系別	應用數學系	考試時間	2月9日（星期三）第四節

注意事項：

- 作答時，請於答案卷上標明題號，並請勿任意更改題目符號，且請詳列過程，只有答案不給分。請盡量清楚完整回答你會的問題，不要只是每題回答一小部份。
- 本試題共有 **5** 個問題，總計 **100** 分。

1. (20 %) Let T be an arbitrary linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$. Find an expression for $T(x,y,z)$ where $(x,y,z) \in \mathbb{R}^3$. Justify your answer.

2. (20 %) Let V be a finite-dimensional vector space and let $T\colon V \to V$ be a linear transformation. Suppose that $\mathrm{R}(T) \cap \mathrm{N}(T) = \{\mathbf{0}\}$. Prove that $V = \mathrm{R}(T) \oplus \mathrm{N}(T)$.

3. (20 %) Let V be the vector space of polynomial functions in two real variables x and y of degree at most 2. Let $\beta = \{1, x, y, x^2, y^2, xy\}$ be an ordered set for V.
   (a) Show that β is a basis for V.
   (b) Let T be a linear operator on V defined by
   $$T(f(x,y)) = \frac{\partial}{\partial x} f(x,y) + \frac{\partial}{\partial y} f(x,y).$$
   (c) Let A be the matrix representation of T in the ordered basis β. Find A.
   (d) Let A be the matrix in part (c), find a Jordan canonical form J and an invertible matrix Q such that $J = Q^{-1}AQ$.

4. (20 %) Let A be an m-by-n matrix with entries in $\mathbb{F}$, where $\mathbb{F}$ is either $\mathbb{R}$ or $\mathbb{C}$. Let $A^* = \overline{A^T}$. Show that $\operatorname{rank}(A^*A) = \operatorname{rank}(A)$.

5. (20 %) Let $W = \{(x,y,z) \mid x+y+z=0\}$ be a subspace of $V = \mathbb{R}^3$. Let $P\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the projection onto W along the subspace $L = \{t(1,2,-1) \mid t \in \mathbb{R}\}$. Find an expression for $P(x,y,z)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。