考試科目	微積分	系所別	應用數學系	考試時間	2月9日(三)第三節

Show all your work and carefully justify all your answers.
Answers without explanation will not revive any score.

1. (24%) Evaluate the integrals.

    A. (8 %) $\displaystyle\int \frac{\arcsin(\ln x)}{x}\,dx$

    B. (8%) $\displaystyle\int_1^\infty \left[\frac{x}{1+x^2} - \frac{1}{x}\right]\,dx$

    C. (8%) $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx$

2. (10%) Evaluate the integral $\displaystyle\iiint_E xy\,dV$, where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.

3. (10%) Evaluate the integral $\displaystyle\iint_R e^{x+y}\,dA$, where R is given by the inequality $|x| + |y| \le 1$.

4. (8%) If f is continuous and $\displaystyle\int_0^9 f(x)\,dx = 4$, find $\displaystyle\int_3^0 xf(x^2)\,dx$.

5. (10%) Find the length of the curve $y = \sqrt{x-1}$ from $x = 1$ to $x = \frac{5}{4}$.

6. (10%) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = \frac{1}{x} + \frac{1}{y}$ subject to the constraint $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

7. (10%) Find the limit

$$\lim_{x\to 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3}.$$

8. (8%) Determine whether the series

$$\sum_{n=1}^\infty (-1)^{n-1} n^{-1/3}$$

is conditionally convergent, absolutely convergent, or divergent.

9. (10%) Use the $\epsilon - \delta$ definition of the limit to show that $\displaystyle\lim_{x\to 1}(x^2 - 2x) = -1$.

備　　　註	一． 作答於試題上者，不予計分。 二． 試題請隨卷繳交。