國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：微積分【財管系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

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※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

For each question, clearly show all derivation steps and explicitly state the calculus rules, identities, or formulas used. Answers without sufficient justification may not receive full credit.

Q1. Please calculate the following limit: (10 points)

$$\lim_{x\to 0^+} x^{\tan x}$$

Q2. Please calculate the following limit: (10 points)

$$\lim_{y\to 0} \left(\sqrt{y^4+y^2+4}-\sqrt{y^4+y^2}\right)$$

Q3. Please solve the following question: (10 points)

$$\int_0^1 \int_{\sqrt{s}}^{\sqrt{1+s}} \frac{X}{\int_0^1 \left(\sqrt{1+t}-\sqrt{t}\right)\,dt}\,dy\,ds$$

Q4. Please solve the following question: (10 points)

$$\int_0^1 \int_0^x \frac{Z}{\int_0^1 \int_0^t e^s\,ds\,dt}\,e^y\,dy\,dx$$

Q5. Please evaluate the following integral. (10 points)

$$\int_0^x \cos(x)\left[\cos\left(\frac{x}{2}\right)\right]^2 dx-\frac{1}{4}x$$

Q6. Please solve the following question: (10 points)

$$\int \frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}\,dx$$

Q7. Please solve the following question: (10 points)

Given a bond with the following characteristics:

- Bond price: $P=96$; Modified duration: $D=4$; Convexity: $C=24$

Recall that a second-order Taylor expansion of a function $f(x)$ around $x=0$ is given by:

$$f(x)\approx f(0)+f'(0)x+\frac{1}{2}f''(0)x^2.$$

Consider two possible policy scenarios announced by the Federal Open Market Committee (FOMC):

- Scenario A (Rate cut): the interest rate decreases by 50 basis points.
- Scenario B (Rate hike): the interest rate increases by 50 basis points.

(1) Using a first-order Taylor expansion, estimate the bond price under each scenario. (4 points)

(2) Using a second-order Taylor expansion, write down the approximation formula to re-estimate the bond price under each scenario. Briefly explain why the second-order term leads to different effects under a rate cut versus a rate hike. (6 points)

Q8. Please solve the following differential equation: (10 points)

$$y' + y = xy^2$$

Q9: Please solve the following question: (10 points)

Due to recent political instability in a major oil-producing country, the daily profit (in million USD) of a representative oil-exporting firm becomes uncertain.

Let x denote the daily oil price fluctuation index, where $0 \le x \le 2$.
The profit function of the firm is given by

$\pi(x) = 20 - 4x^2$.

Assume that x is uniformly distributed on [0, 2].

(1) Please compute the expected daily profit of the firm. (5 points)
(2) Please briefly interpret the economic meaning of your result. (5 points)

Q10. Please solve the following question: (10 points)

Consider a representative listed stock whose one-period return (in percentage points) depends on a market sentiment variable x, where $0 \le x \le 1$.

Suppose the conditional expected return of the stock is given by
$E(R \mid x) = 20 - 4x^2$.

However, market sentiment x itself is uncertain and follows the density function
$f(x) = 2x$, $0 \le x \le 1$.

(1) Compute the unconditional expected return of the stock. (6 points)
(2) Briefly explain why the unconditional expected return differs from $E(R \mid x = 0)$. (4 points)