靜宜大學 101 學年度碩士班暨碩士在職專班招生考試試題

學系：財務與計算數學系　　科目：線性代數

（請一律於答案卷作答）

選擇題（每題 6 分，共 24 分）

(1) For $A=\begin{bmatrix}1 & -1\\ 2 & -2\end{bmatrix}$, find vector b so that there are infinitely many solutions(無窮多組解) for system $Ax=b$.(a) b=$\begin{bmatrix}1\\ -2\end{bmatrix}$ (b) b=$\begin{bmatrix}1\\ 0\end{bmatrix}$ (c) b=$\begin{bmatrix}0\\ 1\end{bmatrix}$ (d)以上皆非.

(2) For $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -4 & 0\end{bmatrix}$, the dimension of null space of A is

(a) 1 (b) 2 (C) 3 (d)以上皆非.

(3) For S = vector space spanned by $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}3\\5\\3\end{bmatrix},\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}0\\-1\\0\end{bmatrix}\right\}$, the dimension of S is

(a) 2 (b) 3 (c) 4 (d)以上皆非.

(4) For $A=\begin{bmatrix}1 & 0 & 4 & -1 & 0\\ 0 & -1 & 1 & 2 & 0\\ 0 & 0 & 2 & 4 & 0\end{bmatrix}$, rank(A) = (a) 2 (b) 3 (c) 5 (d)以上皆非.

非選擇題

(1)~(4) 為是非題，請判斷敘述為 True(是)或 False(非)。將答案依題目序號寫於非選擇題之答案紙上（是非題每格 3 分）

(1)_______ : For any A $m\times n$ real matrices, we have $rank(A)=rank(A^T)$.

(2)_______ : For any A be a $n\times n$ real matrix, $\det(A)\neq 0 \Leftrightarrow A^{-1} exists$.

(3)_______ : For any A be a $n\times n$ real matrix, $rank(A)=n \Leftrightarrow A^{-1} exists$.

(4)_______ : For any A , B ,C be $n\times n$ real matrices,
we have $A(B-C)=0 \Leftrightarrow B=C$.

(5)~(12) 為計算題及證明，請寫出詳細計算及證明過程，否則不予計分。將答案依題目序號寫於非選擇題之答案紙上（計算題及證明題每格 8 分）

(5) Find all the eigenvalues of $A=\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$. (i.e.,求 A 的特徵值)

(6) For $A=\begin{bmatrix}1 & -1\\ 2 & -2\end{bmatrix}$, find vector b so that there are infinitely many solution for system $Ax=b$

(7) Use Gauss Elimination for augmented matrix to solve system of equations as following

$$\begin{aligned}-x_1-2x_3&=-1\\ 3x_1-2x_2+x_3&=-2\\ x_1-3x_2-x_3&=-2\end{aligned}$$

.(i.e.,以高斯法求解)

(8) $A=\begin{bmatrix}1 & 2 & 0 & -1 & 3\\ 0 & 2 & 1 & 2 & 0\\ -2 & -2 & 1 & 4 & -6\end{bmatrix}$, find rank(A) (i.e.,求 A 的秩).

(9) For $A=\begin{bmatrix}1 & 2 & 0 & -1 & 0\\ 0 & 1 & 1 & 2 & 0\\ 0 & 0 & 0 & 1 & 1\end{bmatrix}$ Find the bases of Col (A) = column space of A and Null(A) = null space of A . (i.e., 求 A 的行空間基底 及 列空間基底)

(10) For S = vector space spanned by $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}0\\3\\2\end{bmatrix},\begin{bmatrix}-1\\2\\1\end{bmatrix}\right\}$,

find the dimension of S. (i.e.,求 S 的維度)

(11) Find the determinant of $A=\begin{bmatrix}1 & 0 & 0 & 0\\ -1 & 2 & 0 & 1\\ 1 & 1 & 0 & 0\\ 0 & 0 & -1 & 1\end{bmatrix}$.(i.e. 求 det(A) = ?)

(12) For any A be a $n\times n$ real matrix, prove that if A is invertible then $\det(A)\neq 0$.
(i.e. 證明： 若 A 為可逆，則 $\det(A)\neq 0$)