

## 第一部分：財金數學（共六十分）

## 注意事項：

※※答案請於答案卷(本)上標示清楚，如 ANS: \_\_\_\_\_ ※※

※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

A. 多重選擇題（每題的選項中至少有一個是不正確的；每題必須全部選對才給分；答錯不倒扣。）

1. (8 points)

Which of the following statements are incorrect?

- (A) For a square matrix  $A$ , a linear system  $Ax = b$  has a unique solution if and only if  $\det(A) \neq 0$ . In that case, the rows of  $A$  are linearly independent and span the entire space  $\mathbb{R}^n$ .
- (B) If  $M$  is a real symmetric matrix (e.g., a covariance matrix), then the Spectral Theorem guarantees that  $M$  is diagonalizable by an orthogonal matrix and all eigenvalues of  $M$  are real.
- (C) In the eigen-decomposition  $A = V\Lambda V^{-1}$ , if a matrix  $A$  is defective (i.e., has fewer linearly independent eigenvectors than its size), it can still be diagonalized provided every eigenvalue has algebraic multiplicity greater than one.
- (D) A linear map  $T: V \rightarrow W$  is an isomorphism if and only if it is both one-to-one and onto, which requires  $\dim(V) > \dim(W)$ .
- (E) For matrices  $A$  and  $B$  with compatible dimensions, the rank satisfies
- $$\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$$

2. (8 points)

You are a quantitative analyst validating a risk management system. The system uses a Geometric Brownian Motion (GBM)  $S$  to model equity prices. The equity dynamics are defined as  $\frac{dS}{S} = \mu dt + \sigma dB$  and define the one-period gross return

$$R_{t,1} = \frac{S_{t+1}}{S_t}.$$

Which of the following statements are incorrect?

- (A)  $E[R_{t,1}] = e^\mu$
- (B)  $E[R_{t,1}] = e^{\mu - \frac{1}{2}\sigma^2}$
- (C)  $\text{Var}(R_{t,1}) = e^{2\mu}(e^{\sigma^2} - 1)$
- (D)  $\text{Var}(R_{t,1}) = e^{2(\mu - \frac{1}{2}\sigma^2)}(e^{\sigma^2} - 1)$
- (E)  $\frac{S_{t+1}}{S_t}$  follows lognormal distribution.

3. (8 points)

A portfolio manager chooses nonnegative exposures  $x_1$  and  $x_2$  to two trading strategies:

- Strategy 1 contributes +1 unit of expected excess return per unit exposure.
- Strategy 2 contributes +1 unit of expected excess return per unit exposure.

So expected excess return is:  $\max_{x_1, x_2} f(x_1, x_2) = x_1 + x_2$ .

Risk control is an (elliptical) quadratic risk budget:  $2x_1^2 + x_2^2 = 12, x_1 \geq 0, x_2 \geq 0$ .

Which of the following statements are incorrect?

- (A) The optimal exposures are  $x_1^* = \sqrt{2}, x_2^* = 2\sqrt{2}$ , the risk constraint binds, and  $\lambda^* = \frac{1}{\sqrt{2}}$ .
- (B) Because the objective is increasing in both exposures, the optimal point must be on the boundary; the optimum is  $x_1^* = 0, x_2^* = \sqrt{12}$ .
- (C) The Lagrangian has a global “ $x$ -max,  $\lambda$ -min” saddle point.
- (D) If shorting were allowed (i.e., dropping  $x_1 \geq 0, x_2 \geq 0$ ), the optimal solution would change.

B. 計算題 (每題都必須附上計算過程)

4. (6 points)

$$\int_{-1}^1 x^3 e^x dx = \underline{\hspace{4cm}}$$

5. (10 points)

Let  $X = \{\text{apple, grape, pineapple}\}$ .

- (1) (6%) Explain why the collection of sets  $C = \{\{\text{grape}\}, \{\text{grape, pineapple}\}, \{\text{apple, grape}\}\}$  is a basis for a topology on  $X$ .
- (2) (4%) What is the topology generated by  $C$ ?

6. (8 points)

Solve the following system of first-order linear differential equations for  $y_1(t)$  and  $y_2(t)$ , given the initial conditions  $y_1(0) = 3$  and  $y_2(0) = 2$ .

$$\begin{cases} \frac{dy_1}{dt} = 5y_1 - 2y_2 \\ \frac{dy_2}{dt} = y_1 + 2y_2 \end{cases}$$

7. (12 points)

Suppose there are two non-dividend-paying stocks  $X$  and  $Y$ . Their prices today are

$$X_0 = 18, Y_0 = 22.$$

One year from now, the economy can be in one of three states. The stock prices at  $t_1$  are:

State $i$	$X_1(i)$	$Y_1(i)$
1	30	12
2	20	22
3	28	14

- (1) (6%) Consider a contingent claim  $C = (C_1, C_2, C_3)^T$ , where  $C_i$  is the payoff in state  $i$ . If  $C$  can be replicated using only stocks  $X$  and  $Y$ , then  $(C_1, C_2, C_3)$  must satisfy a linear restriction. Find this restriction.
- (2) (6%) Under the no-arbitrage assumption, find the one-year risk-free rate  $r$ .

**第二部分：財務管理（共四十分。第 8-12 題，每題 8 分）**

**注意事項：**

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**※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。**

**※※一律作答於所附之考試答案卷(本)上。若於試題卷上作答者，將不予計分※※**

8. (8 points)

- a. You are evaluating a mine that will produce \$10,000 worth of ore in the first year. Due to extraction difficulty, the value of the ore will decline at a rate of 8% per year forever. The appropriate interest rate is 6%. Calculate the current value of this mining operation.
- b. You have a newborn child and need to fund a four-year college education starting in 18 years. Currently, costs average \$12,500 per year and are expected to increase by 4% annually. Your savings account earns 7% interest. Calculate the total amount required on the child's 18th birthday to pay for all four years of tuition (paid at the start of each school year).

Which of the following correctly identifies the current value of the mine and the total savings required for college at age 18?

Option	Mine Operation Value	College Savings Required
(A)	\$166,666.67	\$84,330.12
(B)	\$71,428.57	\$97,110.07
(C)	\$125,000.00	\$101,224.45
(D)	\$71,428.57	\$89,145.30
(E)	\$92,592.59	\$97,110.07

9. (8 points) The Aether Electronics Corporation is considering an investment in a new high-speed drone docking station project that has an estimated life of four years. The initial cost of the specialized assembly equipment is \$180,000, and the machine will be depreciated straight-line over its four-year life to a residual value of \$0.

The docking station project is expected to result in sales of 800 units in year one. Sales volume is estimated to grow by 12% per year through year four. The price per unit that Aether Electronics will charge its customers is \$420 and is expected to remain constant throughout the project's life. These units have a variable cost to manufacture and ship of \$190 each.

The expansion of manufacturing capacity will require an increase in various net working capital accounts. It is estimated that the Aether Electronics Corporation needs to hold 3% of its annual sales in cash, 7% of its annual sales in accounts receivable, and 12% of its annual sales in inventory, while maintaining accounts payable at 8% of its annual sales. The firm is in the 21% tax bracket and has a weighted average cost of capital (WACC) of 11%.

Based on the financial projections provided for Aether Electronics Corporation, what is the Net Present Value (NPV) of the docking station project?

- (A) \$317,824.48
- (B) \$361,358.67
- (C) \$385,612.90
- (D) \$408,398.67
- (E) \$425,110.25

10. (8 points) Exactly one year ago, Stellar Capital purchased a position in "Quantum-Tech" 10-year bonds at a price reflecting a 6.0% yield to maturity (YTM). The bonds have a \$1,000 par value and an 8.0% annual coupon.

Today, after receiving the first annual interest payment, the fund is selling the bonds. Due to a market shift, the required YTM for these bonds is now 5.0%. Stellar Capital is subject to a 15% flat tax rate on all investment income (both coupons and capital gains).

Calculate the after-tax annual total rate of return for this investment.

- (A) 8.45%
- (B) 9.12%
- (C) 10.82%
- (D) 12.15%

11. (8 points) A senior analyst has issued a research note on "Advanced-Chipset Inc", which is currently trading at a market spot price of \$100.00 per share. Based on their internal 12-month projections, the firm expects the following:

- Capital gains: The price target for the 12-month horizon is projected at \$105.00.
- Income generation: The stock is forecasted to issue a cash dividend of \$1.00 per share at the end of the 12-month holding period.

Market Context and Risk Metrics:

- Equity Risk Premium: 600 basis points.
- Risk-Free Rate: 4.50%.
- Beta: 1.80.

Determine the alpha of this position to justify a "Strong Sell" recommendation.

- (A) -9.30%
- (B) -11.80%
- (C) -14.25%
- (D) 6.00%
- (E) 15.30%

12. (8 points) You are a senior analyst at Apex Investment Partners. You are evaluating "Horizon Tech Solutions" (HTS) to determine its intrinsic value and growth profile. You have gathered the following financial data:

Variable	Value
Risk-free Rate ( $r_f$ )	3.5%
Expected Return on the Market ( $E(r_m)$ )	9.5%
Horizon Tech Beta ( $\beta$ )	1.25
Expected Earnings Per Share ( $EPS_1$ )	\$5.00
Expected Return on Equity (ROE)	15%
Earnings Retention (Plowback) Ratio ( $b$ )	50%

Please calculate the intrinsic value of HTS today ( $V_0$ ) and determine what percentage of that value is represented by the present value of growth opportunities (PVGO). Choose the option that most closely matches your results.

- (A)  $V_0 = \$45.45$ ; PVGO% = 0.00%
- (B)  $V_0 = \$62.50$ ; PVGO% = 12.50%
- (C)  $V_0 = \$71.43$ ; PVGO% = 36.36%
- (D)  $V_0 = \$83.33$ ; PVGO% = 45.45%
- (E)  $V_0 = \$90.00$ ; PVGO% = 50.00%