

I Multiple-Answers (Choose ALL that apply; full credit only — no partial credit)

1. (20 points) On the Oakley Planet, a household chooses quantities of two goods: a food composite x and household water w , under drought-related pricing and policy changes. Assume preferences are **continuous, locally nonsatiated**, and (when needed) **convex**. Unless stated otherwise, $x \geq 0$ and $w \geq 0$. Let prices be $p_x > 0$, $p_w > 0$, and income $m > 0$. Assume solutions exist whenever a demand object is referenced. Answer Questions (a) and (b) below.

(a) (10 points) Choose all statements that are **always true**:

- A. The Slutsky substitution matrix is symmetric.
- B. The Slutsky substitution matrix is negative semidefinite.
- C. If a good is Giffen at (p_x, p_w, m) , then it must be inferior at (p_x, p_w, m) .
- D. If a good is inferior at (p_x, p_w, m) , it must be Giffen at (p_x, p_w, m) .
- E. If both goods are normal at (p_x, p_w, m) , then each uncompensated own-price effect is negative.
- F. Under quasilinear utility $u(x, w) = x + \phi(w)$ (with an interior solution in x), compensated and uncompensated demand for w are identical for any change in prices.

(b) (10 points) Choose all statements that are **always true**:

- A. The budget set $B(p_x, p_w, m) = \{(x, w) \geq 0 : p_x x + p_w w \leq m\}$ is nonempty, closed, and bounded.
- B. With locally nonsatiated preferences, any optimal choice must satisfy $p_x x + p_w w < m$.
- C. If preferences are locally nonsatiated and strictly convex, the optimal bundle (if exists) is unique.
- D. If utility is differentiable and the optimum is interior, then at the optimum $MRS_{xw} = \frac{MU_x}{MU_w} = \frac{p_x}{p_w}$.
- E. Marshallian demand $x(p_x, p_w, m)$ is homogeneous of degree 0 in (p_x, p_w, m) .
- F. If p_x increases (holding p_w and m fixed), then the Marshallian demand for x must weakly decrease.

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II Problem Set

2. (30 points) On the Oakley Planet, the Oakley Water Authority introduces an increasing block tariff to conserve water. A household chooses food x and household water w . Income is $m = 60$. Preferences are:

$$u(x, w) = x + 40 \ln(w)$$

Food is the numeraire with price $p_x = 1$ and $w > 0$. Water is priced via an increasing block tariff: the first L units cost 1 per unit; any additional units cost 3 per unit. Thus, the water bill is:

$$B_L(w) = \begin{cases} w, & \text{if } 0 < w \leq L \\ L + 3(w - L), & \text{if } w > L \end{cases}$$

- (a) (6 points) Construct the household's budget constraint using $B_L(w)$, and substitute it into the utility function to write utility as a function of w only.
- (b) (8 points) Let $L = 10$. Solve for the optimal bundle (x_1, w_1) . Clearly indicate which price block is relevant at the optimum and verify that your solution lies in that region.
- (c) (8 points) A drought policy reduces the low-price allowance from $L = 10$ to $L = 6$. Solve for the new optimal bundle (x_2, w_2) .
- (d) (8 points) Compute the compensating variation (CV), in dollars, for the policy change from $L = 10$ to $L = 6$, and provide an interpretation in your own words.

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3. (20 points) Consider a monopolist, Firm M, who faces two potential consumers: Consumer 1 and Consumer 2. The firm's marginal cost of production is zero, and it seeks to maximize total revenue. The table below shows the marginal willingness to pay (WTP) for each unit of the product for both consumers. Assume the product units are indivisible and resale is not possible.

Quantity	Marginal WTP (\$)	
	Consumer 1	Consumer 2
1st Unit	10	11
2nd Unit	9	5

Please answer the following questions:

- (a) (5 points) If Firm M is restricted to setting a single uniform unit price P (Linear Pricing) and cannot distinguish between consumers, what is the optimal price P^* ? What is the firm's total profit?
- (b) (5 points) Suppose Firm M can perfectly distinguish between Consumer 1 and Consumer 2 and charge them different unit prices, P_1 and P_2 . What are the optimal prices for each consumer? What is the total profit?
- (c) (5 points) Suppose Firm M adopts a "Pure Bundling" strategy, selling only a package containing "2 units" of the product. Individual units are not sold. What is the optimal bundle price P_B ? Is this strategy strictly better than simple monopoly pricing?
- (d) (5 points) Suppose the firm offers a menu of contracts for consumers to self-select:
- **Option A:** Buy 2 units for a total price of T_2 .
 - **Option B:** Buy 1 unit for a price of T_1 .

The firm wants to design a mechanism where Consumer 1 chooses Option A and Consumer 2 chooses Option B (Separating Equilibrium). Write down the Participation Constraints (IR) and Incentive Compatibility Constraints (IC), and determine the profit-maximizing values for T_1 and T_2 .

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4. (30 points) Consider an industry with three firms: Firm 1, Firm 2, and Firm 3. They produce a homogeneous good with the inverse market demand function:

$$P(Q) = 120 - Q$$

where $Q = q_1 + q_2 + q_3$. The firms have asymmetric cost structures:

- Firm 1 (Efficient): $TC_1(q_1) = 0$.
- Firm 2 and Firm 3 (Inefficient): $TC_i(q_i) = 30q_i$ for $i = 2, 3$.

Please answer the following questions:

- (a) (10 points) Find the Cournot-Nash equilibrium quantities (q_1^*, q_2^*, q_3^*) and the profit for each firm.
- (b) (10 points) Suppose all three firms form a Cartel to maximize the **joint industry profit**. They agree to distribute the production quotas to minimize total costs and then split the profit.
1. What are the optimal production quantities for each firm (q_1^C, q_2^C, q_3^C) ?
 2. What is the total industry profit?
- (c) (10 points) Consider a sequential game where firms move one after another:
- **Stage 1:** Firm 1 chooses q_1 .
 - **Stage 2:** Firm 2 observes q_1 , then chooses q_2 .
 - **Stage 3:** Firm 3 observes q_1 and q_2 , then chooses q_3 .

Find the Subgame Perfect Equilibrium quantities.

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