註 1:本試題禁止使用任何類型計算機

註 2:每題均需寫出詳細解題過程，僅寫出最終答案者不給分

1. Solve the following ordinary differential equations (ODEs)
   (a) (10%) $y'' + y' - 2y = 3e^x + \sin(x)$
   (b) (10%) $(y')^2 = 2yy''$

2. (a) (5%) Find the inverse Laplace Transform of $F(s) = ln\left(\frac{s+1}{s-1}\right)$
   (b) (10%) Solve the initial value problem (IVP) using the Laplace Transform
   $$y'' - 2y' - 3y = t^2\delta(t-2) \quad \text{where } y(0) = 1 \text{ and } y'(0) = 0$$
   *Note: $\delta(t-2)$ is a Dirac Delta function.

3. (15%) Find the general solution for the following ODE in terms of series solution in powers of x and identify two linearly independent solutions. Also, determine the respective convergence radius of each identified independent solution. For each solution, unless it can be written as a known function or it is a polynomial, you at least need to explicitly write out first three non-zero terms.
   $$(x^2 - 1)y'' + 2xy' - 12y = 0$$

4. (10%) Find the double Fourier sine series of $3x + y^2$, $0 \le x \le 1$, $0 \le y \le 2$.

5. (10%) Find the **complex** Fourier series expansion of the given periodic function $f(t)$
   $$f(t) = e^{-t} \quad -1 < t < 1$$

6. (15%) Solve the 2-dimensional wave equation,
   $$\frac{\partial^2 u}{\partial t^2} = 4\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \quad (0 < x < 2\pi,\ 0 < y < 2\pi,\ t > 0)$$
   $$u(0, y, t) = u(2\pi, y, t) = u(x, 0, t) = u(x, 2\pi, t) = 0$$
   $$u(x, y, 0) = 0,\ \frac{\partial u(x,y,0)}{\partial t} = 1$$

7. (15%) Solve for the temperature distribution, $u(x, t)$, of semi-infinite bar.
   $$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (x > 0,\ t > 0)$$
   $$u(x, 0) = 0,\ u(0, t) = A \text{ (constant temperature)}$$

Note: the following integrals are of your information.

$$\int x \sin(ax)\, dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a}$$

$$\int x \cos(ax)\, dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a}$$

$$\int x^2 \sin(ax)\, dx = \left(\frac{2x}{a^2}\right)\sin(ax) + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos(ax)$$

試題隨卷繳回