1. (5 points) Comment on the following statement concerning the Central Limit Theorem.

   "For independent and identically distributed random variables with finite mean μ and variance σ^2, the sample average $\bar{X}$ converges in distribution to a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$ as the sample size increases."

   You need to provide an explanation for why the statement is TRUE or FALSE.

2. (15 points) Let $(X_1, X_2, X_3) \sim^{i.i.d.} N(0, \sigma^2)$.

   (a) Define
   $$Y = \frac{X_1}{|X_2|}$$
   Find the distribution of Y.

   (b) Define
   $$W = \frac{X_1 - X_2}{X_1 + X_2}$$
   Find the distribution of W.

   (c) Consider the following two statistics:
   $$U = (X_1 + X_2 + X_3)^2$$
   and V is the range of the random sample (X_1, X_2, X_3). Show that U and V are independent.

3. (15 points) Let $(X_1, \ldots, X_n)$ be a random sample from $U[0, \theta]$, $\theta > 0$.

   (a) Find the method of moments estimator for θ, denoted by $\hat{\theta}$. Is $\hat{\theta}$ a consistent estimator for θ?

   (b) Use $\hat{\theta}$ to propose a test statistic to test
   $$H_0 : \theta = 9 \text{ vs. } H_1 : \theta \neq 9$$
   at significance level $\alpha = 0.05$. Specify the rejection region.

   (c) Given the realization of the random sample $(X_1, X_2, X_3) = (6.30, 2.28, 7.46)$, construct a 95% confidence interval for θ, and determine whether you can reject the null hypothesis at significance level $\alpha = 0.05$ in part (b).

4. (15 points) Let $(X_1, \ldots, X_n)$ be a random sample from a continuous distribution with distribution function $F(x)$ and probability density function $f(x)$. Assume that F is invertable. Define $X_{(n)} = \max(X_1, \ldots, X_n)$, and
   $$Y_n = n(1 - F(X_{(n)}))$$
   Find the limiting distribution of Y_n.

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5. (25 points) Consider the model

$$logy = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$
$$E(u|x) = 0, u|x \sim N(0, \sigma^2)$$

where $logy = \log(y)$. We run the OLS regression of $logy$ on x_1 and x_2, and obtain the predicted value of $logy$ for any value of the independent variables:

$$\widehat{logy} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2.$$

Let $\hat{\sigma}$ be a consistent estimator of σ, and $\hat{u}_i$ be the residual term for $i = 1, ..., N$.

(a) Find $E(\exp(u))$.

(b) Find $E(y|x)$.

(c) Find a consistent estimator for the predicted value of y, $\hat{y}$, in terms of $\widehat{logy}$ and $\hat{\sigma}$.

(d) If we know that u is not normally distributed, find a consistent estimator for the predicted value of y, $\hat{y}$, in terms of $\widehat{logy}$ and $\hat{u}_i$.

6. (10 points) Suppose that you are interested in estimating the ceteris paribus relationship between y and x_1. For this purpose, you collect data on y and x_1 and two control variables x_2 and x_3. Let $\tilde{\beta}_1$ be the simple regression estimate from y on x_1 and let $\hat{\beta}_1$ be the multiple regression estimate from y on x_1, x_2 and x_3.

(a) If x_1 is almost uncorrelated with x_2 and x_3 in the sample, and x_2 and x_3 are highly correlated, would you expect $\tilde{\beta}_1$ and $\hat{\beta}_1$ to be similar or very different? Explain.

(b) If x_1 is highly correlated with x_2 and x_3, and x_2 and x_3 have small partial effects on y, would you expect $se(\tilde{\beta}_1)$ or $se(\hat{\beta}_1)$ to be smaller? Explain. ($se(.)$ is standard error)

7. (15 points) Consider the probit model $P(y = 1|z_1, z_2, q) = \Phi(z_1\delta_1 + \gamma_1 z_2 q)$, where z_2 is a continuous variable, q is independent of z_1 and z_2, $q \sim N(0,1)$, and $\Phi(.)$ is c.d.f. of $N(0,1)$. Moreover, z_1 and z_2 are observed but q is not.

(a) Find $\frac{\partial P(y=1|z_1,z_2,q)}{\partial z_2}$.

(b) Find $P(y = 1|z_1, z_2)$.

[Some useful $N(0,1)$ probabilities]

$$P(N(0,1) \le 2.33) = 0.99, \quad P(N(0,1) \le 1.96) = 0.975$$
$$P(N(0,1) \le 1.64) = 0.95, \quad P(N(0,1) \le 1.28) = 0.90$$

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