

※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. (Total 20 points) General Equilibrium and Endowment Economy

Consider a two-period overlapping generations model in a closed, endowment economy.

- Endowments: Each individual receives y_1 when young and y_2 when old.
- Preference: $u(c_1) + \beta u(c_2)$, where c_1 denotes consumption when young and c_2 denotes consumption when old. The utility function satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$, and $\beta \in (0, 1)$.
- Demographics: The population grows at a constant rate n .
- Markets: There is a competitive credit market where agents borrow or lend at a market-clearing interest rate r .

There is no government or production sector. In answering the following questions, focus on the steady state and omit time subscripts.

- (a) (5 points) State the equations characterizing the competitive equilibrium (c_1, c_2, r) . Explain your results.
- (b) (5 points) Analytically determine and intuitively explain the effect of an increase in future endowment (y_2) on the equilibrium interest rate (r).
- (c) (5 points) Now consider the same economy with a government sector operating a Pay-As-You-Go social security program. Each young individual pays a lump-sum tax τ . Each old individual receives a benefit b . The government maintains a balanced budget in every period. State the equations characterizing the competitive equilibrium (c_1, c_2, r, b) . Explain your results.
- (d) (5 points) Compare the equilibrium interest rate r of this economy with the previous one (without social security). Which is higher? Provide an intuitive explanation.

2. (Total 30 points) Labor Supply and Balanced Growth Equilibrium

Time is indexed by $t = 0, 1, 2, \dots$. In each period, the household solves a static problem. There is no saving or borrowing, so the household consumes its labor income each period. Specifically, the representative household solves the following problem in each period:

$$\max_{C_t, L_t} U(C_t, L_t),$$

subject to the budget constraint

$$C_t = W_t L_t,$$

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and the time constraint,

$$0 \leq L_t \leq 1,$$

where C_t is consumption, L_t is hours worked, and W_t is the wage, which follows the exogenous process:

$$W_t = (1 + g_w)W_{t-1}.$$

Thus, the economy is characterized by an exogenous wage sequence $\{W_t\}_{t=0}^{\infty}$, and the household's choices generate sequences $\{C_t, L_t\}_{t=0}^{\infty}$.

- (a) (6 points) As written, the wage process is exogenous, so the model is partial equilibrium. Provide one economic environment in which the same setup can be interpreted as a general equilibrium framework. You may introduce additional agents if needed. In your answer, explain how the labor market clears in that environment.
- (b) (5 points) Given the preference specification

$$U(C_t, L_t) = \begin{cases} \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, & \text{if } \sigma \neq 1, \\ \ln(C_t) - \psi \frac{L_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, & \text{if } \sigma = 1, \end{cases}$$

where $\sigma > 0$, $\psi > 0$, and $\nu > 0$. Assume an interior solution. Derive the conditions characterizing the optimal choices C_t and L_t .

- (c) (6 points) Using your answer above, show that the growth rates of consumption and hours worked are constant over time (i.e., independent of t). For this question, you may use the log-difference approximation for growth:

$$g_{x,t} \approx \log X_t - \log X_{t-1}.$$

- (d) (8 points) Using your answer above, determine whether hours worked increase, decrease, or remain constant over time. How does the value of σ govern the relative strength of the income and substitution effects in this economy? Discuss how these two effects compete to determine the path of L_t as wages grow.
- (e) (5 points) Suppose the economy must reach a "Balanced Growth Path" where hours worked L_t remain strictly interior and constant as $t \rightarrow \infty$. Based on your previous answers, What restriction on σ is implied? Explain your logic.

3. (Total 30 points) Unemployment and Job Search

Consider an economy where every worker participates in the labor force, meaning each individual is either employed or unemployed. We normalize the total labor force being 1, and define U as the fraction of workers currently unemployed, while the remaining $1 - U$ represents those who are employed. The equilibrium depends on the individual decision-making process of workers faced with potential job offers.

The utility an individual derives from accepting a job at wage w can be expressed as:

$$u_e(w) = \sqrt{w} - 0.05 \quad (1)$$

Conversely, should the individual choose to reject an offer and remain unemployed, they receive a utility level determined by:

$$u_u(b) = b + 1 \quad (2)$$

Here, b represents the government-provided unemployment benefits.

The probability that an unemployed worker receives a job offer is p , where the associated wage w is drawn from a uniform distribution such that $w \sim \text{Uniform}[0, \bar{w}]$.

- (a) (5 points) Specify the condition under which the individual finds it optimal to accept a job offer at wage w .
- (b) (5 points) Define the reservation wage w^* that makes an individual indifferent between accepting the offer and remaining unemployed, and express this condition mathematically. Given the parameters $b = 4.95$, solve for the equilibrium value of w^* .
- (c) (5 points) Define $H(\bar{w}) \equiv \Pr(w \geq \bar{w})$ as the probability for the wage offer to be larger than \bar{w} . Write the functional form of $H(\bar{w})$ for the uniform distribution $w \sim \text{Uniform}[0, \bar{w}]$ under $\bar{w} = 100$.
- (d) (5 points) Assume that in each period, a fraction λ of employed workers lose their jobs, while a fraction p of unemployed workers receive a wage offer. Write the equations for the flows into and out of the unemployment pool, expressing the job-finding rate in terms of p and $H(w^*)$.
- (e) (5 points) Solve for the steady-state unemployment rate U using the parameter values provided in part (b) and assuming $\lambda = 0.1$ and $p = 1$.

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(f) (5 points) If b increases, explain the impact on w^* , $H(w^*)$, and U . Use a graph to assist your answer.

4. (Total 20 points) The Malthusian Model of Growth

"In this economy, the entire population is in the labor force. Production requires land (L) and labor force (N_t): $Y_t = zL^\alpha N_t^{1-\alpha}$. All output is consumed ($C_t = Y_t$), and the standard of living per capita is $c_t = C_t/N_t$. The birth (b) and death (d) rates depend on c_t where $b(c_t) = c_t/2$ and $d(c_t) = 0.1 - c_t/5$.

- (a) (5 points) Given the birth rate and death rate, write down the law of motion for the total population in the economy. Show how N_{t+1} depends on N_t and c_t .
- (b) (5 points) Express consumption per worker, c_t , as a function of productivity z and land-per-worker (L/N_t).
- (c) (5 points) Provide a mathematical and conceptual definition of steady state total population N^* . Let $z = 1/7$, $L = 100$, $\alpha = 0.5$. Find N^* and c^* . Explain the intuition for why the population converges to N^* .
- (d) (5 points) Suppose productivity z increases to 1, use a figure to show the transitional dynamics for N^* and c^* .

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