國立嘉義大學 101 學年度
應用數學系碩士班（甲組）招生考試試題

科目：線性代數

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」。

1. Compute the symmetric LDL^T factorization of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. (10%)

2. Find the determinant of $A=(a_{i,j})$, if $a_{i,j}=i+j$ for $i,j\in\{1,2,\cdots,n\}$. (10%)

3. Let A be an $m\times n$ matrix over ℝ, prove that $rank(A)\le m$ and $rank(A)\le n$. (10%)

4. Prove that $A=\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ are similar. (10%)

5. Let V be a vector space of dimension 2. We say that a linear map $T:V\to V$ is self-adjoint if $<T(v),w>=<v,T(w)>$.
   (1) Suppose that $\{v_1,v_2\}$ is an orthonormal basis for V and $T:V\to V$ is a self-adjoint linear map. Show that the matrix of T relative to that basis is symmetric. (15%)
   (2) Assume that $f:V\times V\to R$ is a map defined by $f(v,w)=<T(v),w>$ where T is a self-adjoint linear map. Show that f is a bilinear symmetric form of V. (15%)

6. Let $A=\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{pmatrix}, y=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Find $\min_{x\in\mathbb{R}^2}\|Ax-y\|$. (15%)

7. In $\mathbb{R}^4$, let $w_1=(1,0,1,0)$, $w_2=(0,1,0,1)$, $w_3=(1,0,0,0)$. Prove that $\{w_1,w_2,w_3\}$ is linearly independent. Use the Gram-Schmidt process to compute the orthogonal vectors v_1, v_2 and v_3 such that $span\{w_1,w_2,w_3\}=span\{v_1,v_2,v_3\}$ and then normalize these vectors to obtain an orthonormal set. (15%)