國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	統計學	系所別	經濟學系	考試時間	2 月 5 日（四）第四節

Instructions:

- Please label the question numbers and answer them in numerical order.
- A table of the cumulative standard normal distrbution is provided on the last page.
- You must show your work and provide complete answers for full credit. Answers with no explanations or derivations, or incorrect ones, will receive a score of zero.

1. (30%) Suppose X and Y have the following joint probability density function (p.d.f.):

$$f_{XY}(x, y; \mu) = \begin{cases} \frac{\mu^2}{2} \exp\left(-\mu x - \frac{\mu y}{2}\right) & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

for $\mu > 0$, where $\exp(\cdot)$ denotes the exponential function.

(a) (5%) What is the coverage probability of the interval $\left[X, X + \frac{Y}{2}\right]$ for μ?

(b) (5%) Suppose $\{(X_i, Y_i)\}$, $i = 1, \ldots, n$, is a random sample from the joint p.d.f. $f_{XY}(x, y; \mu)$. Find the maximum likelihood estimator $\hat{\mu}_{ML}$ for μ.

(c) (5%) Find the Cramér-Rao Lower Bound for unbiased estimators of μ.

(d) (5%) Is $\hat{\mu}_{ML}$ the minimum variance unbiased estimator? Why or why not?

(e) (10%) Find the limiting distribution of $\sqrt{n}(\hat{\mu}_{ML} - \mu)$ as $n \to \infty$.

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2. (45%) For positive random variables X and Y, suppose

$$Y = \beta X + \sigma(X)e$$

with an unknown parameter β, $\sigma(X) = X$, $\mathbb{E}[X] = 1$, $\text{Var}(X) = 2$, $\mathbb{E}[e \mid X] = 0$, and $\mathbb{E}[e^2 \mid X] = \sigma^2$. To estimate β, you are given a random sample $\{(X_i, Y_i)\}, i = 1, \ldots, n$. Let $\overline{X}$ and $\overline{Y}$ denote the respective sample means. Consider the three estimators:

$$W_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{Y_i}{X_i}, \quad W_2 = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2}, \quad W_3 = \frac{\sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} X_i}$$

(a) (5%) Find $\mathbb{E}\left[\frac{Y}{X}\right]$.

(b) (5%) Find $\mathbb{E}\left[\overline{Y} \mid X_1, \ldots, X_n\right]$.

(c) (5%) Find $\text{Var}(\overline{Y})$.

(d) (6%) Are W_1, W_2, and W_3 unbiased estimators for β?

(e) (6%) Are W_1, W_2, and W_3 consistent estimators for β?

(f) (6%) Are W_1, W_2, and W_3 efficient estimators for β?

(g) (5%) Let $X = \log(Z)$, where log denotes the natural logarithm. Interpret β in terms of the association between Y and Z.

(h) (7%) Following (g), suppose the parameter of interest, γ, is the *average* marginal effect of a one-unit change in Z on the expected value of Y. Given a random sample $\{(Y_i, Z_i)\}, i = 1, \ldots, n$, consider the estimator:

$$W_4 = \frac{W_2}{\overline{Z}}$$

where $\overline{Z}$ denotes the sample mean of Z_i. Will W_4 tend to overestimate, underestimate, or consistently estimate γ? (Suppose $\beta > 0$.)

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3. (25%) The table below reports OLS and Probit regression results of children's economic outcomes on child and parental characteristics, where standard errors are in parentheses and the variables are described below:
   - Income: Child's income in thousand dollars.
   - College Grad: a dummy variable equal to one if the adult child graduated from a four-year college and zero otherwise.
   - Male: a dummy variable equal to one if the child is male and zero if female.
   - Mother Drinks: a dummy variable equal to one if mother drinks alcohol and zero otherwise.
   - Mother's Education and Father's Education: years of education of parents.
   - Log Parents' Income: the natural logarithm of parents' income in dollars.

	(1)	(2)	(3)
	OLS	OLS	Probit
Dependent Variable	Income	College Grad	College Grad
Mother's Education		0.021 (0.008)	0.060 (0.019)
Father's Education		−0.004 (0.007)	−0.010 (0.017)
Log Parents' Income		0.011 (0.027)	0.008 (0.064)
Male	−5.200 (0.590)	−0.159 (0.041)	−0.398 (0.090)
Mother Drinks	−7.800 (8.500)		
Male × Mother Drinks	−2.800 (5.100)		
Constant	75.800 (10.200)	0.766 (0.264)	0.142 (0.566)
Observations	874	897	1088

(a) (5%) Interpret all of the coefficient estimates in Column (1).

(b) (5%) Define $\theta = \mathbb{E}[\text{Income} \mid \text{Male} = 1, \text{Mother Drinks} = 1] - \mathbb{E}[\text{Income} \mid \text{Male} = 1, \text{Mother Drinks} = 0]$. Use the coefficient estimates in Column (1) to provide a point estimate of θ and describe how to obtain its standard error.

(c) (5%) Consider a female child whose parents each have 16 years of education and whose parents' log income is 12 (in dollars). Using Column (3), compute the probability that the child graduates from college.

(d) (5%) Using Column (3), what is the difference in predicted probabilities of graduating from college for the female child in (c) compared with a male child with all other characteristics identical?

(e) (5%) Now use Column (2) to estimate the change in predicted probabilities for the comparison in (d).

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Table 2 CUMULATIVE NORMAL DISTRIBUTION

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\,dt$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\Phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\Phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001

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