國立臺灣科技大學

114學年度碩士班招生

試題

系所組別：1200自動化及控制研究所碩士班

科　　目：工程數學

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(總分為 100 分；所有試題務必於答案卷內頁依序作答)

1. **(10%)** Solve the following first-order ordinary differential equation with initial values.

$$y' + 3y = 5e^{2x} - 6; y(0) = 2$$

2. **(10%)** Write the general solution for the following second-order ordinary differential equation.

$$y'' - 2y' + 10y = 0$$

3. **(10%)** Compute the following Laplace transform. f has period of 6, and

$$f(t) = \begin{cases} 5 & \text{for } 0 < t \leq 3 \\ 0 & \text{for } 3 < t \leq 6 \end{cases}$$

4. **(12%)** Find the recurrence relation and use it to generate the first five terms of a power series solution about 0.

$$y' - xy = 1 - x$$

5. **(8%)** Determine whether the vectors are linearly independent or dependent in the appropriate R^n.

$$\mathbf{2i},\ 3\mathbf{j},\ 5\mathbf{i}\text{-}12\mathbf{k},\ \mathbf{i}+\mathbf{j}+\mathbf{k} \qquad \text{in } R^3$$

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6. (**15%**) Consider a matrix **A**

$$\mathbf{A}=\begin{bmatrix}1 & -3\\ 2 & -4\end{bmatrix}$$

(1) (**8%**) Find the eigenvalues and the corresponding eigenvectors of **A**.

(2) (**7%**) Use Cayley-Hamilton theorem to compute $\mathbf{A}^{50}$.

7. (**15%**) Write the Fourier cosine and sine series for $f(x)$ on the interval. Determine the sum of each series.

$f(x) = 1-x^3,\ 0\le x \le 2$

8. (**10%**) Use the Fourier transform to solve the following differential equation.

$y'' + 6y' + 5 = \delta(t-3)$

9. (**10%**) Consider the following complex equation, find u and v so that $f = u + iv$. Determine whether the Cauchy-Riemann equations hold for all points (x, y) and determine whether f is differentiable for all z.

$f(z) = z^2 - iz$