

國立中山大學 114 學年度 碩士班考試入學招生考試試題

科目名稱：微積分【財管系碩士班甲組】

—作答注意事項—

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 1 頁

Q1: Please calculate the following limit: (10 points)

$$\lim_{x \rightarrow 0^+} x^{\sin(x)}$$

Q2: Please calculate the following limit: (10 points)

$$\lim_{y \rightarrow 2} (\sqrt{y^4 + y^3 - y + 1} - \sqrt{y^4 - y^2 + y})$$

Q3: Please solve the following question: (10 points)

$$\int_0^1 \int_{\sqrt{x}}^{\sqrt{1+x}} y^3 e^{y^2} dy dx$$

Q4: Please solve the following question: (10 points)

$$\int_0^1 \int_0^x e^{x+y} dy dx$$

Q5: Please solve the following question: (10 points)

$$\int \sin^3(5x) \cos^2(5x) dx$$

Q6: Please find the general solution: (10 points)

$$y' + 3y = e^x \cos(x)$$

Q7: The weekly demand of a product is modeled by the probability density function as follows:

$$f(x) = \frac{(-x^2 + 6x)}{46}, 0 \leq x \leq 6, \text{ where } x \text{ is the number of units sold in (thousands)}$$

Please calculate the probability that a randomly chosen week will have sales between 1000 and 3000 units. (10 points)

Q8:

- (1) Considering that bond price (P) depends on interest rate (r), please construct the Taylor expansion (up to the second order) to estimate the price change (dP) based on interest rate change (dr). (5 points)
- (2) On the assumption that the short-term treasury bill price is 100, the current interest rate is 5%, the modified duration is 5, and the convexity is 20. Please use the Taylor expansion to calculate the new Treasury bill price based on the following information released by the Federal Open Market Committee in September. (5 points)

"In light of the progress on inflation and the balance of risks, the Committee decided to lower the target range for the federal funds rate by 1/2 percentage point..."

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Q9: A Monopolist produces one type of output for two types of customers. Customers of type 1 are willing to pay a price of $100-10A$ dollars per unit when it produces A units for them. In the case of type 2 customers, if it produces B units, they are willing to pay for the units at a price of $80-4B$ dollars each. Considering that the total cost of producing $A+B$ units is $100+10(A+B)$. What is the optimal number of units of A and B to be produced in order to maximize profits? (10 points)

Q10: Let's suppose you are a mutual fund manager and you want to know whether some Dow 30 stocks in the financial service industry suit your investment goals. You hope that the standard deviation of this portfolio is less than 25% based on the weights of two stocks.

- the weight of Goldman Sachs (GS): 40%;
- the weight of American Express (AXP): 60%;

A matrix of Variance-Covariance of these two stocks is shown below.

	GS	AXP
GS	0.06	0.03
AXP	0.03	0.04

- (1) Please calculate the portfolio variance using the vector and square matrix (7 points)
- (2) According to your answer above, are the two stocks' weights adequate for your needs, and why? (3 points)