國立成功大學

114學年度碩士班招生考試試題

編　號：128

系　所：電機工程學系

科　目：線性代數

日　期：0210

節　次：第 3 節

注　意：1.不可使用計算機
　　　　2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (40 pts, 4 pts each) Mark each statement True or False (2 pts for correct answer). Justify each answer (2 pts).

   a. Suppose that **u**, **v**, and **w** are vectors in $\mathfrak{R}^n$. If **u** is orthogonal to **v** + **w**, then **u** is orthogonal to **v** and **w**.

   b. Similar to the above condition in (a), If $\|\mathbf{u} - \mathbf{v}\| = \mathbf{0}$, then **u** = **v**.

   c. The set of 2×2 matrices that contain exactly two 1's and two 0's is a linearly independent set in $M_{2\times2}$.

   d. If {**u**, **v**} is a linearly dependent set, then each vector is a scalar multiple of the other.

   e. If V is a subspace of $\mathfrak{R}^n$ and W is a subspace of V, then $W^\perp$ is a subspace of $V^\perp$.

   f. If **u** is in the row space and the column space of an $n\times n$ matrix A, then **u** = **0**.

   g. If v_1, v_2, and v_3 come from different eigenspace of A, then it is impossible to express v_3 as a linear combination of v_1 and v_2.

   h. If A is diagonalizable and invertible, then A^{-1} is diagonalizable.

   i. There is no square matrix A such that $\det(AA^T) = -1$.

   j. If $\det(A) = 0$, then A is not expressible as a product of elementary matrices.

2. (30 pts, 10 pts each) Let $A = \begin{bmatrix} \dfrac{1}{3} & \dfrac{1}{4} \\ \dfrac{2}{3} & \dfrac{3}{4} \end{bmatrix}$.

   a. Find a matrix P such that $P^{-1}AP$ is diagonal.
   b. Let $n \geq 1$ be an arbitrary integer. Find A^n.
   c. Use the result of part (b) to determine the matrix $B = \lim_{n\to\infty} A^n$.

3. (30 pts) Let A be the matrix given by $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$ with rank $A = 2$.

   a. Find an SVD of A.   (12 pts)
   b. Compute A^+.   (6 pts)
   c. Find a least-squares solution for $Ax = b$, where $b = [1, 0]^T$.   (6 pts)
   d. Find the least-squares error for part (c). (6 pts)