國立臺灣師範大學 114 學年度碩士班招生考試試題

科目：**工程數學**　　適用系所：**電機工程學系**

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Please solve the following differential equation (20 points)

$$(2xy + 3y)dx + (4y^3 + x^2 + 3x + 4)dy = 0, \quad y(0) = 1$$

2. Apply Laplace transform to solve the following initial value problem: (20 points)

$$y''(t) - 4y'(t) + 4y(t) = \delta(t-1), y(0) = 0, y'(0) = 1$$

where $\delta(t)$ is the Dirac delta function.

3. Factorize the following into LU decomposition, i.e., product of lower and upper triangular matrix. The diagonal lines of the lower triangular matrix should be all ones (20 points).

$$\begin{bmatrix} 2 & 4 & 3 & 5 \\ -4 & -7 & -5 & -8 \\ 6 & 8 & 2 & 9 \\ 4 & 9 & -2 & 14 \end{bmatrix}$$

4. Are the following statements true or false? (40 points)

1) if the inverse of a matrix **A** exists, its reduced row-echelon form is an identity matrix **I**
2) For any row elementary matrice, its inverse is also an row elementary matrice.
3) The determinants of a matrix **A** and its row-echelon form are identical.
4) The dimension of a vector space **V** is always infinite.
5) For a vector set *S* with m>n vectors, it can span a vector space *V* with dimension n. Then, vectors in *S* must be dependent.
6) For a non-square matrix **A**, either its columns or rows are linearly dependent.
7) Even with det(**A**)=0, the matrix **A** could still be full rank in some cases.
8) For any matrix **P** and **Q**, we always have **PQ≠QP**