國立成功大學
114學年度碩士班招生考試試題

編　號：52

系　所：太空與電漿科學研究所

科　目：應用數學

日　期：0210

節　次：第 2 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

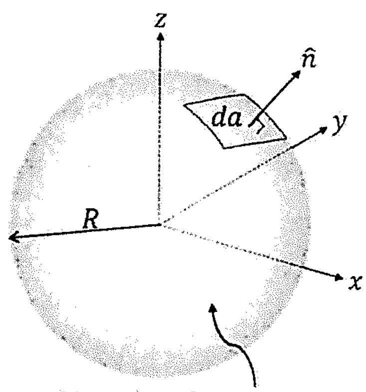
Show your steps clearly for full credit.

1. Solve for each of the following integrals: **(20%)**
   (a) $\int dx \cos^3 x$; (4%)
   (b) $\int dx \cos^2 x$; (4%)
   (c) $\int dx \cos x$; (4%)
   (d) $\int dx \cos^0 x$; (4%)
   (e) $\int dx \cos^{-1} x$. (4%)

2. Given a vector $\mathbf{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ in Cartesian coordinates, where A_x, A_y, and A_z are functions of the coordinates x, y, and z. Calculate $\nabla \cdot (\nabla \times \mathbf{A})$. **(10%)**

3. Let $\mathbf{A} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 0 & a \\ 0 & -a & 2 \end{pmatrix}$, where a is a constant. In regard to the eigenvalues of $\mathbf{A}$, whether real or complex, it is given that $\mathbf{A}$ has only one distinct eigenvalue λ. **(25%)**
   (a) Find λ and give all possible values of a. (10%)
   (b) **For each possible value** of a, let N be the number of independent eigenvectors of $\mathbf{A}$. (Note that N may be different for different values of a.) Then calculate or derive a set of N independent eigenvectors of $\mathbf{A}$ for the corresponding value of a. (15%)

4. Find I by solving the following integral that is integrated over a closed surface S, which is shown in the figure on the right:
$$I = \int_S da\, \hat{n} \cdot \mathbf{F},$$
where da is the differential area, $\hat{n}$ is the unit vector pointing normally outward from the surface,
$$\mathbf{F} = xz^2\hat{x} + \frac{1}{3}y^3\hat{y} + x^2z\hat{z}\ ,$$
and S is a spherical surface represented in Cartesian coordinates by the equation $x^2 + y^2 + z^2 = R^2$. In other words, the spherical surface S is of radius R and has its center at the origin where $(x, y, z) = (0,0,0)$. **(20%)**



Closed surface S:
$x^2 + y^2 + z^2 = R^2$

5. Solve for the functions $f = f(x)$ and $g = g(x)$ in the following set of coupled equations:
$$\begin{cases} \dfrac{df}{dx} = 2g \\ \dfrac{dg}{dx} = 8f \end{cases}$$
with initial conditions $f(0) = a$ and $g(0) = b$, where a and b are constants. **(25%)**