$c = 3 \times 10^8$ meter/second is the speed of light. $\hbar = 1.05 \times 10^{-34}$ Joule·second is the reduced Planck constant.

第一大題 (SECTION 1): 單一選擇題 (Multiple-Choice)
一題五分 (5 points for each question)。請依題號順序作答於答案卡 (Please answer in the order of the question numbers on the answer card)。

1. What is the ground state energy of a particle of mass m in a one-dimensional box of length L?
   (A) $\frac{\hbar^2\pi^2}{2mL^2}$
   (B) $\frac{\hbar^2\pi^2}{8mL^2}$
   (C) $\frac{\hbar^2\pi^2}{mL^2}$
   (D) $\frac{\hbar^2\pi^2}{4mL^2}$
   (E) $\frac{\hbar^2\pi^2}{\sqrt{2}mL^2}$

2. What quantum number determines the overall shape of an atomic orbital?
   (A) Magnetic quantum number
   (B) Spin quantum number
   (C) Principal quantum number
   (D) Angular quantum number
   (E) (C) and (D)

3. If the potential for a particle in one dimension coordinate space x is $V(x)=\lambda x^4$, where λ is a coefficient. Which of the following best describes the behavior of the wave function as $x \to \pm\infty$?
   (A) Approaches zero
   (B) Becomes constant
   (C) Oscillates with increasing amplitude
   (D) Oscillates with constant amplitude
   (E) Not enough information to determine

4. A particle of mass m in a finite potential well with depth V_0 and width L has a ground state energy E_0. Calculate the wavelength of the particle associated with this state.
   (A) $\frac{2\pi\hbar}{\sqrt{2mE_0}}$
   (B) $\frac{2L}{\sqrt{E_0+V_0}}$
   (C) $\frac{2\pi\hbar}{\sqrt{2m(E_0+V_0)}}$
   (D) $\frac{2\pi\hbar}{\sqrt{2mV_0}}$
   (E) $\frac{2L}{\sqrt{\frac{m}{E_0+V_0}}}$

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5. A particle is in a state described by the wave function $\phi(x)=A\exp(-\alpha x^2)$ where x is a special coordinate, and A and α are constants. Calculate the expectation value of x
   (A) $1/(4\alpha)$
   (B) $1/(2\alpha)$
   (C) $1/(8\alpha)$
   (D) 0
   (E) 1

6. What is the expectation value of the angular momentum squared, $\langle L^2\rangle$, for an electron in a p-orbital ($\ell=1$)?
   (A) $2\hbar^2$
   (B) $\hbar$
   (C) $6\hbar^2$
   (D) $3\hbar$
   (E) $\hbar^2$

7. For an electron in an atom, what is the minimum possible value of the total angular momentum J if the orbital angular momentum $L=1$ and the spin $S=1/2$?
   (A) 1/2
   (B) 1
   (C) 3/2
   (D) 2
   (E) −1/2

8. If the mass of a quantum harmonic oscillator is doubled while keeping the force constant the same, what happens to the energy eigenvalues?
   (A) They decrease by a factor of $2^{1/2}$
   (B) They increase by a factor of $2^{1/2}$
   (C) They remain the same
   (D) They decrease by a factor of 2
   (E) They increase by a factor of 2

9. In a system described by a superposition of two orthogonal states $\phi=\alpha\phi_A+\beta\phi_B$, the observables corresponding to ϕ_A and ϕ_B are measured to be A and B, respectively. What must be true for the coefficients α and β if the expectation value $\langle\phi|O|\phi\rangle$ is to be minimized, where O is the observable operator?
   (A) $|\alpha|=|\beta|$
   (B) $|\alpha|=0$ or $|\beta|=0$
   (C) α and β are complex conjugates
   (D) α and β are purely imaginary
   (E) α and β are real

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10. For a particle described by the superposition of two wave functions ϕ_A and ϕ_B, which statement is true regarding the interference term when calculating the probability density?
    (A) The interference term is always zero, as wave functions are orthogonal.
    (B) The interference term is constant and equal to 1.
    (C) The interference term increases the total probability density quadratically.
    (D) The interference term depends on the relative phase between ϕ_A and ϕ_B.
    (E) The interference term is a complex number.

11. In quantum scattering theory, the differential cross-section $d\sigma/d\Omega$ is often calculated. What does this quantity physically represent?
    (A) The probability that a particle scatters per unit area.
    (B) The fraction of scattered particles per unit scattering angle.
    (C) The probability density that a particle scatters into a specific solid angle.
    (D) The total number of particles scattered by the target per unit time.
    (E) The energy transferred to the target per scattered particle.

12. How does special relativity affect the concept of mass?
    (A) Mass decreases as objects move faster.
    (B) Mass remains constant regardless of the speed.
    (C) Mass increases as the speed of an object approaches the speed of light.
    (D) Mass is only relevant in gravitational fields.
    (E) None of the above.

13. If two clocks are synchronized and one moves on a round trip at 0.7c, how much less time will the traveling clock show compared to the stationary clock, if the trip takes 10 years in the stationary frame? Choose the option below that is most accurate.
    (A) 4 years
    (B) 6 years
    (C) 3 years
    (D) 2 years
    (E) 1 year

14. A particle with a mass 2 kg is moving at 0.5c. Find the value below that is closest to its total energy.
    (A) 2.08×10^{17} Joules
    (B) 2.51×10^{17} Joules
    (C) 3.03×10^{17} Joules
    (D) 3.21×10^{17} Joules
    (E) 3.35×10^{17} Joules

15. If two events are separated by 300,000 km and occur 1 second apart in a certain frame, what is their separation in time and space in a frame moving at 0.6c relative to the first, if the events are along the direction of motion?
    (A) 0.3 seconds, 100,000 km
    (B) 0.5 seconds, 150,000 km
    (C) 1.67 seconds, 500,000 km
    (D) 2.0 seconds, 600,000 km
    (E) 2.6 seconds, 800,000 km

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第二大題 (SECTION 2): 填充題 (Fill-in-the-Blank)

一題五分 (5 points for each question)。請依題號順序將答案填於試卷內，無需詳細計算過程 (Please fill in the answers on the answer sheet in the order of the question numbers, without the need for detailed calculations)。

16. A particle with orbital angular momentum quantum number ℓ=3 is in a state described by the magnetic quantum numbers m=2. The z-component of the angular momentum L_z of this state is __(16)__.

17. The zero-point energy of a quantum harmonic oscillator with an angular frequency of 5×10^{14} Hz and a mass of 9.1×10^{-31} kg is __(17)__.

18. & 19. A particle is in a superposition state of the first three eigenstates of a quantum harmonic oscillator with equal amplitudes. The expectation value of the position $\langle x\rangle$ is __(18)__ and the position squared $\langle x^2\rangle$ is __(19)__ for this state. The position operator is $x=\sqrt{\frac{\hbar}{2m\omega}}(a^\dagger+a)$, where m is the particle mass, ω is the oscillator angular frequency, and $a^\dagger$ and a are the creation and annihilation operators, respectively.

20. An observer measures two simultaneous flashes of light 10 km apart in their frame. If these observations are made from a frame moving at 0.8c relative to the light sources, the distance between these events in the stationary source frame is __(20)__.

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