

國立臺灣海洋大學114學年度碩士班考試入學招生考試試題

考試科目：微積分

學系組名稱：運輸科學系碩士班不分組

*可使用本校提供之計算器

1. 答案以橫式由左至右書寫在答案卷上。2. 請依題號順序，並標示題號作答。

1. (10%) Find the area of the region completely enclosed by the graphs of the functions

$$f(x) = x^3 - 3x + 3 \text{ and } g(x) = x + 3.$$

2. (10%) Evaluate the improper integral $\int_{-\infty}^{\infty} x e^{-x^2} dx$

3. (10%) Find the second-order partial derivatives of the function $f(x, y) = e^{xy^2}$

4. (10%) Determine the minimum value of $f(x, y) = x^2 + xy + y^2$ subject to the constraint $x + y = 20$

5. (10%) Determine the average value of the function $f(x, y) = 3x^2y$ over the rectangular Region defined by $2 \leq x \leq 5$, $1 \leq y \leq 4$.

6. (10%) Give the function $f(x) = \sqrt{19 - x}$, the point $c = 10$ and the positive number $\varepsilon = 1$. Find $L = \lim_{x \rightarrow c} f(x)$. Then find a number $\delta > 0$ such that for all x .

7. (10%) Find the point on the line $\frac{x}{a} + \frac{y}{b} = 1$ that is closest to the origin.

8. (10%) Using the partial derivative definition to find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(0, 0)$ if

$$f(x, y) = \begin{cases} \frac{\sin(x^3 + y^4)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

9. (20%) Find the limits: (a) $\lim_{x \rightarrow 1} \frac{\sqrt{2x(x-1)}}{|x-1|}$ (b) $\lim_{x \rightarrow 0^+} \frac{(\ln x)^2}{\ln(\sin x)}$