

國立高雄第一科技大學 101 學年度 碩士班 招生考試 試題紙

系所別：電腦與通訊工程系

組別：通訊組

考科代碼：1211

考科：機率

注意事項：

- 1、本科目得使用本校提供之電子計算器。
- 2、請於答案卷上規定之範圍作答，違者該題不予計分。

不可使用字典

1. Suppose that 10 balls are placed at random into 10 cells, where more than 1 ball is allowed to occupy a cell. What is the probability that all cells are occupied? (10 points)
2. A number  $x$  is selected at random in the interval  $[-2, 2]$ . Let the events  $A = \{x > 0\}$ ,  $B = \{|x - 1| < 0.5\}$ , and  $C = \{x < 0.75\}$ . Based on the information, find the probabilities of  $B$ ,  $A \cap B$ , and  $A \cap C$ . (15 points)
3. A voltage  $X$  is uniformly distributed in the set  $\{-4, -3 \dots 2, 3\}$ . Find the mean and variance of  $Y = \cos(\pi X/8)$ . (10 points)
4. The joint pdf (probability density function) of two random variables  $X$  and  $Y$  is defined as

$$f_{XY}(x, y) = \begin{cases} cxy, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

where  $c$  is a constant.

- (a) Find  $c$ . (10 points)
- (b) Find the marginal densities  $f_X(x)$  and  $f_Y(y)$ . (10 points)
- (c) Find  $P(X \leq 1)$ . (5 points)

5. Let  $X$  be a random variable with a Gaussian pdf with mean  $m$  and standard deviation  $\sigma$ :

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2}, \quad -\infty < x < \infty.$$

Let  $Y = 5X + 2$ .

- (a) Find  $f_Y(y)$ , that is the pdf of  $Y$ . (10 points)

- (b) Let  $m = 0$  and  $\sigma = 1$ , find  $\int_{-\infty}^2 f_Y(y) dy$ . (10 points)

6. The random variables  $X$  and  $Y$  have the joint pdf

$$f_{XY}(x, y) = e^{-(x+y)}, \quad \text{for } 0 < y < x < 1.$$

Find the pdf of  $Z = X + Y$ . (10 points)

7. A discrete-time random process is defined by  $X_n = s^n$  for  $n \geq 0$  where  $s$  is selected at random from the interval  $(0, 1)$ . Sketch a sample path of the process, and find the pdf of  $X_n$ . (10 points)