國立臺北大學114學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　　目：數理統計

□可　☑不可使用計算機

Part I

1. Let X and Y be two random variables with the joint probability density function
$$f(x,y) = 2(x+y), \quad 0 \le x \le y \le 1$$
   (a) (10%) Let $Z = X + Y$. Find the probability density function of Z.
   (b) (10%) Let $W = \ln\left(\frac{Y}{X}\right)$. Find the probability density function of W.

2. Let Y be uniformly distributed on the interval (0, 1). Conditionally on Y=y, let X be uniformly distributed on the interval (0, y).
   (a) (5 %) Find the probability density function of X.
   (b) (10%) Find E(X) and Var(X).
   (c) (5 %) Find P(X < 0.3).

3. Let $f_{xy}(x,y)$ be the joint probability function of two random variables X and Y. Suppose that the joint moment generating function is
$$M_{(X,Y)}(t_1,t_2) = E(e^{t_1X+t_2Y}) = \left[\frac{1}{3}(e^{t_1+t_2}+1)+\frac{1}{6}(e^{t_1}+e^{t_2})\right]^2$$
   (a) (5 %) Find the joint probability function of X and Y.
   (b) (5 %) Find the marginal distribution of X and the marginal distribution of Y.

Part II

1. Let $X_1, X_2, \dots, X_n$ be a random sample from Binomial(k,θ), where k is known and θ is unknown.
   (a) (5 %) Find a complete sufficient statistic of θ.
   (b) (5 %) Find the uniformly minimum-variance unbiased estimator (UMVUE) of
$$\tau(\theta) = P_\theta[X_1 = 1].$$

2. Let $X_1, X_2, \dots, X_n$ be a random sample from a population with pdf
$$f(x|\theta) = \theta x^{\theta-1}\mathbf{1}_{\{0<x<1\}}, \ \theta > 0.$$
   (a) (5 %) Find the method of moments estimator (MME), $\hat{\theta}$.
   (b) (5 %) Find the asymptotic variance of $\hat{\theta}$.
   (c) (5 %) Find a consistent estimator of θ. Prove your claim.

3. Let $\boldsymbol{X}_i = (X_{i1}, X_{i2})', i = 1, \dots, n$, be $i.i.d.$ bivariate normal with unknown $(\mu_1,\mu_2)' = E[\boldsymbol{X}_1]$ and $Var(\boldsymbol{X}_1)$. Let $\theta = \frac{\mu_2}{\mu_1}$ be the parameter of interest ($\mu_1 \neq 0$). Define $Y_i(\theta) = X_{i2} - \theta X_{i1}$.
   (a) (5 %) It is known that $Y_1(\theta), \dots, Y_n(\theta)$ are $i.i.d.$ with $N(K_1, K_2)$. Find K_1 and K_2.
   (b) (5 %) Find a pivotal quantity of θ.
   (c) (5 %) Use the pivotal quantity of (b) to find a $100(1-\alpha)\%$ confidence set, $C(\boldsymbol{X})$, for θ.
   (d) (5 %) Find a condition such that $C(\boldsymbol{X})$ is a finite interval.

4. (5 %) Let $f(x|\theta)$ be the logistic location pdf
$$f(x|\theta) = \frac{e^{(x-\theta)}}{(1+e^{(x-\theta)})^2}, -\infty < x < \infty, -\infty < \theta < \infty.$$
   Based on one observation, X, find the most powerful size α test of $H_0: \theta = 0$ versus $H_1: \theta = 1$.

試題隨卷繳交