國立臺北大學 114 學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　目：統計學

☑可 ☐不可使用計算機

I. 填充題

All values are rounded to 4 decimal points.

1. Let $\{Y_1, \ldots, Y_n\}$ be a random sample from the normal distribution with mean μ and known variance σ^2. The hypotheses are H_0: μ =50 vs H_a: μ >50 and the rejection region $R = \{\bar{y}: \bar{y} > 50 + a\}$. At 0.05 level of significance, the value of a is _____ (5%) and the power of the test for true $\mu = 60$ is _____. (5%) (若無法算出一個值，請用累積密度函數等方式表達)

2. The probability density functions for X and (Y given X) are given below.

   $P_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ *where* $\lambda > 0$ *and* $x = 1,2,\ldots..$

   $P_{Y|X}(y|x) = \binom{x}{y} p^y (1-p)^{x-y}$ *where* $0 < p < 1$ *and* $y = 1,2,\ldots..$

   The probability density function of Y is _____ (5%) and the variance of Y is _____. (5%)

3. Suppose that each of two investments for a portfolio has a 4% chance of a loss of $5 million, a 2% chance of a loss of $1 million, and 94% chance of a gain of $2 million. Two investments are independent of each other. The 10th percentile for the distribution of this portfolio investment return is ____. (5%)

4. Consider a random sample $\{x_1, \ldots, x_n\}$ from the distribution with mean μ and known variance σ^2. Define $\bar{x} = n^{-1}\sum_{i=1}^{i=n} x_i$, $s_1^2 = (n-2)^{-1}\sum_{i=1}^{i=n}(x_i - \mu)^2$, and $s_2^2 = (n-2)^{-1}\sum_{i=1}^{i=n}(x_i - \bar{x})^2$. If n=60, the distributions of $(n-2)s_1^2/\sigma^2$ and $(n-2)s_2^2/\sigma^2$ are _____ and _____, respectively. (10%) (Justify your answers)

5. Consider a population of five weights identical in appearance but weighing 1, 3, 5,7 and 9 ounces. The variance for the population (σ^2) is _____. (5%) Sampling without replacement from the above population with a sample size of 2 produces ten possible samples. Using the ten sample mean values to calculate the variance of $\bar{X}$, it is _____. (5%) Why is this value not the same as σ^2/n? _____ (5%)

II. 問答題

The significance level is 0.05 for all questions in II unless otherwise specified.

1. A college received applications from prospective students. The application form contains a variety of information including gender, the individual's Scholastic Aptitude Test (SAT) score and whether or not the student is in-state. SAT scores are normally distributed. The results are in the following table.

	Sample size	number of students are in-state	SAT score	
			mean	standard deviation
male	36	16	1300	90
female	16	10	1360	130

   (1) Conduct a statistical test to determine whether there is a significant difference between the variances in the SAT score of male and female students. Report the test statistic and make a conclusion (reject or not reject the null hypothesis). (5%)

(2) Based on the result of previous question, conduct a statistical test to determine whether the mean SAT score of male students is significantly LOWER than the mean SAT score of female students. Report the test statistic and make a conclusion (reject or not reject the null hypothesis). (5%)

(3) Are the proportions of in-state students the same for male and female students? Report the test statistic and P-value. (5%)

(4) Construct a 95% confidence interval for the difference of the proportion of in-state students for male and female students. (5%)

2. A researcher wishes to try three different techniques to lower the blood pressure of individual diagnosed with high blood pressure. The subjects are randomly assigned to three groups.The first group takes medication, the second group exercises, and the third group follows a special diet. After four weeks, the reduction in each person's blood pressure is recorded. Assume the blood pressure reduction at each population is normally distributed and that the population variances are equal.

	medication	exercise	special diet
Sample size	4	5	4
mean	10	12	14
Standard deviation	6	26	12

(1) Find the within-group variance. (5%)

(2) To test the claim that there is no difference among the means in blood pressure reduction of three groups, what is the test statistics? (5%)

3. A 10-year study conducted by the American Heart Association provided data on how age, blood pressure, and smoking relate to the risk of stroke. The data are from 20 individuals. Risk is interpreted as the probability (times 100) that the patient will have a stroke over the next 10-year period. For the smoking variable, define a dummy variable with 1 indicating a smoker and 0 indicating a nonsmoker. The regression result is follows.

	coefficient	Standard error
intercept	-91.76	15.22
Age	1.08	0.17
Pressure	0.25	0.05
Smoker	8.74	3.00

From ANOVA table, Sum of Square Residual is 530.21, Sum of Square Total is 4190.95

(1) Conduct a test to determine whether there is at least one significant predictor. Report the test statistic and make a conclusion (reject or not reject the null hypothesis). (5%)

(2) Conduct a test to determine whether smoking a significant predictor for the risk of a stroke. Report the test statistic and make a conclusion (reject or not reject the null hypothesis). (5%)

(3) What percentage of the variability in risk of stroke can be explained by the linear effect of the age, blood pressure and he/she is a smoker or not? (5%)

4. Consider an experiment having five possible outcomes whose probabilities are hypothesized to be equal, i.e., 0.2 for all outcomes. This is to be tested by performing 60 independent replications of the experiment. If the resultant number of times that each of the five outcomes occur is 12, 7, 10, 14 and 17, should the hypothesis be rejected? (5%)

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Standard Normal Distribution (Area in Upper Tail)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

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T distribution

	Area in Upper Tail	
df	0.05	0.025
1	6.314	12.706
2	2.920	4.303
3	2.353	3.182
4	2.132	2.776
5	2.015	2.571
6	1.943	2.447
7	1.895	2.365
8	1.860	2.306
9	1.833	2.262
10	1.812	2.228
11	1.796	2.201
12	1.782	2.179
13	1.771	2.160
14	1.761	2.145
15	1.753	2.131
16	1.746	2.120
17	1.740	2.110
18	1.734	2.101
19	1.729	2.093
20	1.725	2.086
21	1.721	2.080
22	1.717	2.074
23	1.714	2.069
24	1.711	2.064
25	1.708	2.060
26	1.706	2.056
27	1.703	2.052
28	1.701	2.048
29	1.699	2.045
30	1.697	2.042
35	1.690	2.030
40	1.684	2.021
45	1.679	2.014
50	1.676	2.009
55	1.673	2.004

Chi-square distribution

	Area in Upper Tail	
df	0.025	0.05
1	5.024	3.841
2	7.378	5.991
3	9.348	7.815
4	11.143	9.488
5	12.833	11.070
6	14.449	12.592

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F distribution (df1=degree of freedom for numerator, df2=degree of freedom for denominator)

Area in Upper Tail =0.025

		df1								
df2		1	2	3	15	16	19	20	35	36
	10	6.937	5.456	4.826	3.522	3.496	3.435	3.419	3.279	3.274
	11	6.724	5.256	4.630	3.330	3.304	3.243	3.226	3.086	3.080
	12	6.554	5.096	4.474	3.177	3.152	3.090	3.073	2.931	2.925
	13	6.414	4.965	4.347	3.053	3.027	2.965	2.948	2.805	2.799
	14	6.298	4.857	4.242	2.949	2.923	2.861	2.844	2.699	2.694
	15	6.200	4.765	4.153	2.862	2.836	2.773	2.756	2.610	2.605
	16	6.115	4.687	4.077	2.788	2.761	2.698	2.681	2.534	2.529
	19	5.922	4.508	3.903	2.617	2.591	2.526	2.509	2.359	2.353
	20	5.871	4.461	3.859	2.573	2.547	2.482	2.464	2.314	2.308
	35	5.485	4.106	3.517	2.235	2.207	2.140	2.122	1.961	1.955
	36	5.471	4.094	3.505	2.223	2.196	2.128	2.110	1.949	1.942

Area in Upper Tail =0.05

		df1								
df2		1	2	3	15	16	19	20	35	36
	10	4.965	4.103	3.708	2.845	2.828	2.785	2.774	2.678	2.674
	11	4.844	3.982	3.587	2.719	2.701	2.658	2.646	2.548	2.544
	12	4.747	3.885	3.490	2.617	2.599	2.555	2.544	2.443	2.439
	13	4.667	3.806	3.411	2.533	2.515	2.471	2.459	2.357	2.353
	14	4.600	3.739	3.344	2.463	2.445	2.400	2.388	2.284	2.280
	15	4.543	3.682	3.287	2.403	2.385	2.340	2.328	2.223	2.219
	16	4.494	3.634	3.239	2.352	2.333	2.288	2.276	2.169	2.165
	19	4.381	3.522	3.127	2.234	2.215	2.168	2.155	2.046	2.042
	20	4.351	3.493	3.098	2.203	2.184	2.137	2.124	2.013	2.009
	35	4.121	3.267	2.874	1.963	1.942	1.892	1.878	1.757	1.752
	36	4.113	3.259	2.866	1.954	1.934	1.883	1.870	1.748	1.743

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