國立臺北大學 114 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學

☑可 ☐不可使用計算機

1. X is a random variable with $E(X) = 3$, $E(X^2) = 25$. Please calculate the lower bound of the probability Pr(-5 < X < 11). (8 %)

2. The time interval (unit: in minutes) between calls received by a customer service company follows an exponential distribution:

$$f(x) = \frac{1}{3}e^{-x/3}, \ x \geq 0$$

Please use the Poisson distribution to calculate the probability that no calls are received within 6 minutes. (10 %)

3. The density function of X:

$$f_X(x) = \begin{cases} \dfrac{2}{\pi(1+x^2)} & for\ x > 0 \\ 0 & for\ x < 0 \end{cases}$$

Derive the density of Y = lnX . (8 %)

4. Random variable $Y|X = x \sim \text{Uniform}(0, x)$ and $X \sim \text{Uniform}(0,1)$. Please calculate:
   (a) $Cov(X, Y)$. (12 %)
   (b) $Var(Y)$. (12 %)

5. Let X be a random variable of the continuous type that has the probability density function $f(x)$.
   (a) If m is the unique median of the distribution of X and b is a real constant, show that
   $\mathrm{E}(|X-b|) = \mathrm{E}(|X-m|) + 2\int_m^b (b-x)f(x)\,dx$,
   provided that the expectations exist. (10 %)
   (Hint: $\int_{-\infty}^{m}(m-b)f(x)dx + \int_m^{\infty}(b-m)f(x)\,dx = 0$)
   (b) For what value of b is $\mathrm{E}(|X-b|)$ a minimum? (5%)

6. If the probability density function (p.d.f.) of X is $f_X(x) = 2xe^{-x^2}$, $0 < x < \infty$ and zero elsewhere, determine the p.d.f. of $Y = X^2$. (10%)

7. Suppose that $P(A) = 0.7$, $P(A) = 0.5$ and $P([A \cup B]^c) = 0.1$. The superscript c denotes the complement of a set.
   (a) Find P(A ∩ B). (5%)
   (b) Give P(A|B). (5%)

8. A machine shop that manufactures toggle levers has both a day and a night shift. A toggle lever is defective if a standard nut cannot be screwed onto the threads. Let p_1 and p_2 be the proportion of defective levers among those manufactured by the day and night shifts, respectively. We shall test the null hypothesis; H_0: $p_1=p_2$, against a two-sides alternative hypothesis based on two random samples, each of 1,000 levers taken from the production of the respective shifts. (Round the answer to 4 decimal places)
   (a) Define the test statistic and a critical region that has an $\alpha = 0.05$ significance level. (5%)
   (b) If $y_1=37$ and $y_2=53$ defectives were observed for the day and night shifts, respectively, calculate the value of the test statistic and state your conclusion. (5%)
   (c) Find the 95% confidence interval of $p_1 - p_2$. (5%)

試題隨卷繳交