

國立臺灣師範大學 114 學年度碩士班招生考試試題

科目：工程數學

適用系所：機電工程學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) Given the second-order O.D.E. : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$
 - (a) (5 分) Find the general solution of the differential equation.
 - (b) (5 分) If the initial conditions $y(0) = 2$ and $\left.\frac{dy}{dx}\right|_{x=0} = 0$ are given, determine the particular solution.
2. (15 分) Solve the following differential equations:
 - (a) (5 分) Find $\int \frac{x^4+1}{x(x^2+1)^2} dx$
 - (b) (5 分) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
 - (c) (5 分) Solve $y\frac{d^2y}{dx^2} + \left\{\frac{dy}{dx}\right\}^2 + 1 = 0$
3. (15 分) Given a free vibration system, its governing equation: $my'' + cy' + ky = 0$, where m , c & k are constants. Find all possible solutions.
4. (10 分) Solve the following equations:
 - (a) (5 分) Find the inverse Laplace transform of $\frac{s-1}{(s-1)^2+4}$
 - (b) (5 分) Find $\mathcal{L}^{-1}\left\{\frac{s+1}{(s^2+1)(s^2+4s+13)}\right\}$
5. (15 分) Consider a set of functions $\{1, x, 3x^2 - 1\}$ on the interval $-1 \leq x \leq 1$.
 - (a) (5 分) Show that this is an orthogonal set over the interval $[-1, 1]$ with respect to a *unit weight* that is the weight function 1.
 - (b) (10 分) Approximate $\sin(\pi x/2)$ on $(-1, 1)$ by a linear combination of the functions in the set in the sense of least squares.

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6. (20 分) Consider $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$, where $\mathbf{A} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$.

- (a) (5 分) Let $\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}$, where $\mathbf{D}$ is a diagonal matrix. Find $\mathbf{V}$.
- (b) (5 分) Determine the definiteness of the quadratic form, $\mathbf{x}^T \mathbf{A} \mathbf{x}$.
- (c) (5 分) Evaluate $e^{\mathbf{A}}$ that is the matrix exponential of $\mathbf{A}$.
- (d) (5 分) Given $\mathbf{x}(0) = [1 \ 0]^T$, find the state response $\mathbf{x}(t)$ by trying a vector solution of the form: $\mathbf{x}(t) = \xi e^{\lambda t}$, where ξ is a constant vector, and λ is a constant scalar.

7. (15 分) The temperature T at a point (x, y, z) in space is described by $T(x, y, z) = 2x^2 - 2xy + 2y^2 + z^2$.

- (a) (5 分) In which direction from $(2, 1, 1)$ does the temperature T decrease most rapidly?
- (b) (5 分) In which directions from $(2, 1, 1)$ does the temperature T remain constant? Be sure to describe all these directions.
- (c) (5 分) Evaluate $\int_C T ds$ on C defined by $x = \cos t$, $y = \sin t$, and $z = 1$ for t starting from 0 to $\pi/2$. Note that $ds = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$.