

國立臺灣師範大學 114 學年度碩士班招生考試試題

科目：數值分析

適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) Use Taylor expansion to find the rate of convergence of

$$\lim_{h \rightarrow 0} \frac{\sinh h - h \cosh h}{h} = 0.$$

2. Given the convergent order of the following method for solving nonlinear equation $f(x) = 0$.

- (a) (5 分) Newton-Rapbson method.
- (b) (5 分) Fixed point iterative method.
- (c) (5 分) Bisection method.
- (d) (5 分) Secant method.

3. Let $g(x) = \pi + \frac{1}{2} \sin(\frac{x}{2})$ for $x \in [0, 2\pi]$.

- (a) (10 分) Write down the algorithm of the fixed-point iteration for solving the fixed point of g .
- (b) (10 分) Show that the sequence produced the algorithm in part (a) with any initial $x_0 \in [0, 2\pi]$ converges to the unique fixed point of g in $[0, 2\pi]$.

4. (10 分) Let $f(x) = \sin x$ and $x^* = \pi$. Show that Newton's method converges to x^* in order $\alpha = 3$ when x_0 is near x^* .

5. Consider the linear system

$$Ax \equiv \begin{bmatrix} \mu & 1 & 0 \\ \nu & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} \mu + \frac{1}{2} \\ 2 + \nu \\ 2\frac{1}{2} \end{bmatrix}.$$

- (a) (10 分) Find one of values of μ and ν for which A is positive definite.
 - (b) (10 分) Compute the Cholesky factorization of the matrix A in part (a) and apply it to solve the linear system.
 - (c) (10 分) Write down the iteration matrix of Jacobi method for solving the linear system with coefficient matrix A in part (a) and discuss its convergence (convergent or divergent) of Jacobi method.
6. (10 分) Obtain the LU factorization with permutation P of the form $A = P^T LU$ for the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$