

國立高雄大學 114 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構
考試時間：100 分鐘

系所：資訊工程學系
本科原始成績：100 分

是否使用計算機：否

離散數學：

1. Each of the one million car license plates labeled from 000000 to 999999 can be denoted as $x_1x_2x_3x_4x_5x_6$, $0 \leq x_i \leq 9$, $1 \leq i \leq 6$.

(a) (3%) In how many of these car licenses are all the six digits **distinct**?

(b) (3%) In how many of these car licenses are the six digits in the relationship

$$x_1 < x_2 < x_3 < x_4 < x_5 < x_6?$$

(c) (3%) In how many of these car licenses are the six digits in the relationship

$$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6?$$

(d) (3%) In how many of these car licenses does the digit 2 appear?

(e) (3%) In how many of these car licenses does **at least one** of the digits 0, 1, and 2 appear?

(f) (6%) In how many of these car licenses does **at least two** of the digits 0, 1, and 2 appear?

2. (9%) For a set of three distinct numbers, we can order them as $a < b < c$ where **b** is the middle, **a** is the smallest, and **c** is the largest. From this hint, give a combinatorial proof that

$$1 \times n + 2 \times (n - 1) + 3 \times (n - 2) + \cdots + (n - 1) \times 2 + n \times 1 = \binom{n+2}{3}, n \in \mathbb{Z}^+.$$

資料結構：

For questions 3, 4 and 5, please choose the best answer to the question.

3. (5%) Given two sentences below:

1. "Mary is much quicker than John."

2. "John is slightly quicker than Mary."

which data structure is most appropriate to store and analyze the grammatical components (e.g., subject, verb, modifier, adjective, comparison)?

(a) Stack

(b) Queue

(c) Dictionary

(d) Linked List

4. (5%) What is the main advantage of using a double hashing technique in hash tables?

(a) It allows for dynamic resizing of the hash table.

(b) It reduces the clustering problem in open addressing.

(c) It ensures keys are stored in sorted order.

(d) It makes the insertion operation $O(1)$ in all cases.

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5. (5%) Given a binary tree with n nodes, you want to find the maximum sum of values from the root to any leaf. Which of the following approaches is the most efficient?

- (a) Depth-First Search with recursion, using $O(n)$ space for the call stack.
- (b) Breadth-First Search using a queue, with time complexity $O(n^2)$.
- (c) Dynamic programming with bottom-up traversal, using $O(n)$ space and $O(n)$ time.
- (d) Pre-order traversal using a stack, with time complexity $O(n^2)$.

6. You are tasked with storing and searching a set of vectors efficiently. Suppose you have $N=10,000$ vectors, each of size $d=128$. You need to design a system for efficient storage and retrieval of these vectors.

- (a) Propose a data structure to store the vectors, ensuring fast access by a unique identifier for each vector. (10%)
- (b) Describe an algorithm to find the vector most similar to a given query vector using cosine similarity. (10%)
- (c) Analyze the time complexity of your search algorithm. (10%)

7. Given the following set of C codes:

```
#include <stdio.h>
#include <string.h>

void encode(const char input, char* output) {
    int i = 0, j = 0;

    while (input[i] != '\0') {
        char current = input[i];
        int count = 1;

        while (input[i] == input[i + 1]) {
            count++;
            i++;
        }
```

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```
        output[j++] = current;
        output[j++] = count + '0'; // Works for single-digit counts
        i++;
    }

    output[j] = '\0'; // Null-terminate
}

int main() {
    char input[] = "aaabbccccdaa";
    char output[50]; // Fixed size array for output

    encode(input, output);
    printf("Original: %s\n", input);
    printf("Encoded: %s\n", output);
    return 0;
}
```

- (a) Please identify any errors in the provided code. (10%)
- (b) If errors are present, what is the output after correcting them? (10%)
- (c) Describe the objective that this set of codes aims to achieve. (5%)