國立中山大學114學年度
碩士班考試入學招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學114學年度碩士班考試入學招生考試試題

科目名稱：控制系統【電機系碩士班乙組】　　題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　共3頁第1頁

I. Single-Answer Questions: **(15 Points in total)**

1. Consider the state equation: $\dot{x}_1 = x_2, \dot{x}_2 = sinx_1$. In the following answers, which one is not an equilibrium point of the system: **(3 Points)**
   (A) $(-\pi, 0)$. (B) $(0, 0)$. (C) $(\pi, 0)$. (D) $(\pi/2, 0)$. (E) $(10\pi, 0)$.

2. A second order system is represented by the differential equation with the input $u(t)$ and the output $y(t)$ as follows: **(3 Points)**
   $$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} - 5y(t) = \frac{d^2u(t)}{dt^2} + 3u(t).$$
   Use the method of direct decomposition to find the stat space model of this system, which can be represented as
   $$\dot{x}_1 = x_2, \dot{x}_2 = ax_1 + bx_2 + cu; \mathrm{y} = dx_1 + ex_2 + fu.$$
   Determine $g = a + b + c + d + e + f$. (A) g=9. (B) g=10. (C) g=11. (D) g=12. (E) g=13.

3. Given the transfer function $\frac{Y(s)}{R(s)} = \frac{s+1}{s^3+as^2+2s+2}$, determine the value of a so that the system is either **uncontrollable** or **unobservable**. **(3 Points)**
   (A) a=1. (B) a= −1. (C) a=2. (D) a= −2. (E) a=0.

4. Consider a unitary feedback system with the open-loop transfer function $G(s) = \frac{s^3+Ks}{s^4+s^2+1}$. When the system is stable, determine the minimum value b of K. **(3 Points)**
   (A) 1.5<b<1.6. (B) 1.6<b<1.7. (C) 1.7<b<1.8. (D) 1.8<b<1.9. (E) 1.9<b<2.

5. For the unity feedback system with the open-loop transfer function $G(s) = \frac{1}{s^2+s}$, apply a feedforward PD controller $G_c(s) = c + ds$ such that the dominant poles of the closed-loop system are located at $-1 \pm j\sqrt{5}$. **(3 Points)** Find $e = c - d$. (A) e=1. (B) e=2. (C) e=3. (D) e=4. (E) e=5.

II. Multiple-Choice Questions with Multiple Answers: **(15 Points in total)**

6. Consider a unitary feedback system with the open-loop transfer function $G(s) = \frac{-s^4-5s^2}{s^7+5s^5+4s^3-4}$. Suppose the closed-loop system has L, I, and R poles on the left-hand side, the imaginary axis, and the right-hand side of the complex plane, respectively. **(3 Points)** (A) L+I=5. (B) L+R=3. (C) R+I=5. (D) I-R=2. (E) I-L=2.

7. Consider a second order system with the transfer function $G(s) = \frac{1}{s^2+bs+c}$. Applying the input
   $$u = cos^2(t),$$
   the system produces an unbounded output. Find all possible c. **(3 Points)**
   (A) c=0. (B) c=1. (C) c=2. (D) c=3. (E) c=4.

8. Consider a unitary feedback system with the open-loop transfer function $G(s) = \frac{1}{s(s^2+s+a)}$. Let GM and PM be its gain margin and phase margin, respectively. In the following answers, choose the right statements. **(3 Points)** (A) When a<1, the system is stable. (B) GM= $\mathbf{-20log_{10}a}$. (C) GM=$\mathbf{20log_{10}a}$. (D) When PM=45°, a>1.54. (E) When PM=45°, a<1.54.

9. For a LTI (linear time-invariant) system, choose the right statements from the following answers. **(3 Points)** (A) When it is asymptotically state under zero input (ZIAS), it is BIBS (bounded-input bounded-state). (B) When it is BIBO (bounded-input bounded-output), it is BIBS. (C) When it is ZIAS, it is BIBO. (D) When it is BIBS, it is ZIAS. (E) When it is BIBS, it is BIBO.

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10. For the signal flow graph shown in Figure 1 where $G_1 = G_2 = G_3 = G_4 = G_5 = G_6 = H_1 = H_2 = 1$, let $Y_1 = aY_2, Y_3 = bY_2, Y_4 = cY_2, Y_5 = dY_2, Y_6 = eY_2$. Choose the right answers from the following statements: **(3 Points)** (A) a=3. (B) b=2. (C) c=1. (D) d=1. (E) e=2.

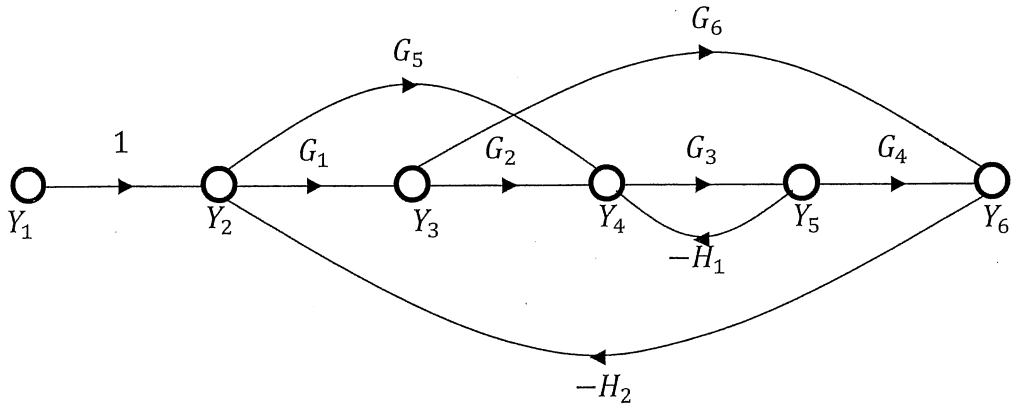

Figure 1.

III. The transfer function of a system is given by $G(s) = \frac{1}{(s-2)(s-3)(s-7)}$. **(20 Points in total)**

(a) Employ the direct decomposition method to find a state space representation of the system. **(2 Points)**

(b) Is the system controllable and observable? If yes, please check your answers. **(3 Points)**

(c) With $x_1 = y$ (the output), $x_2 = \dot{x}_1$ and $x_3 = \dot{x}_2$. design a state control feedback $u = -Kx + v$ with $K = [K_1 \quad K_2 \quad K_3]$ so that with $v = 0$, the characteristic roots of the closed-loop system are located at $s = -3 \pm j$ and $s = -5$. **(5 Points)**

(d) By applying the Luenberger observer: $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$ to estimate the real state x, design the observe gain $L = [L_1 \quad L_2 \quad L_3]^T$ so that the roots of the characteristic polynomial $\det(sI - (A - LC))$ of the error system are located at $s = -1 \pm j$ and $s = -3$. **(5 Points)**

(e) Using the dynamic output feedback $u = -K\hat{x} + v$ with K as in (c) where $\hat{x}$ is a solution of $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$ with L as in (d), find the transfer function of the new closed-loop system (from v to y). **(5 Points)**

IV. A unitary feedback system has the open-loop transfer function $G(s) = \frac{K}{s(s+1)}$. **(15 Points in total)**

(a) Find the minimum value b of K such that under a ramp input $r(t)=t$, the steady state error is less than or equal to 0.05. **(2 Points)**

(b) With $K = 2b$, find the phase margin. **(3 Points)**

(c) With $K=2b$, design a feedforward phase-lead compensator to meet the following specifications: Phase margin $\geq 30°$; Steady state error ≤ 0.03. **(10 Points)**

V. Consider the nonlinear system: $\dot{x}_1 = x_2, \dot{x}_2 = cosx_1$. **(10 Points in total)**

(a) Find the all equilibrium points. **(2 Points)**

(b) Find the linearized system at each equilibrium point. **(3 Points)**

(c) Determine the stability property for each linearized system. **(5 Points)**

VI. Draw the root locus for the closed-loop system shown in Figure 2. **(15 Points in total)**

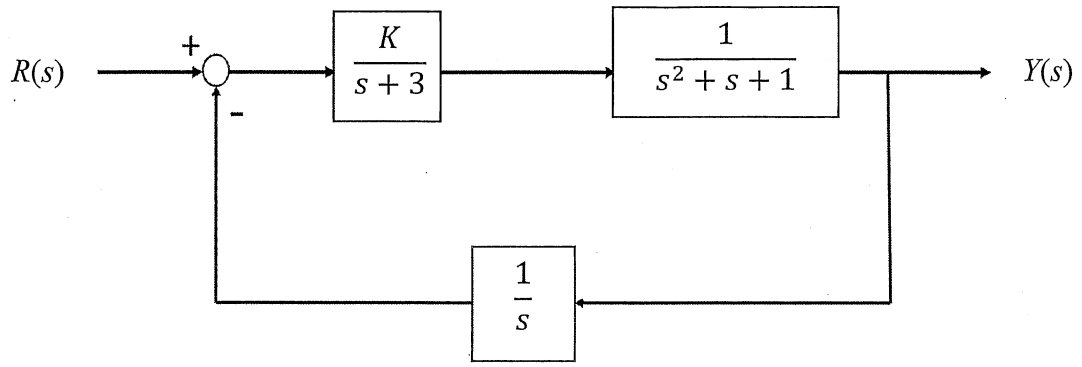

Figure 2: The block diagram considered in problem VI.

VII. Consider the feedback system shown in Figure 3. **(10 Points in total)**

(a) Find the values of K and a to satisfy the following frequency domain specifications: $M_r = 1.5$ and $\omega_r = 10$ rad/sec . **(5 Points)**

(b) For the values of K and a determined in part (a), calculate the peak percent overshoot of the step response and the bandwidth of the closed-loop system. **(5 Points)**

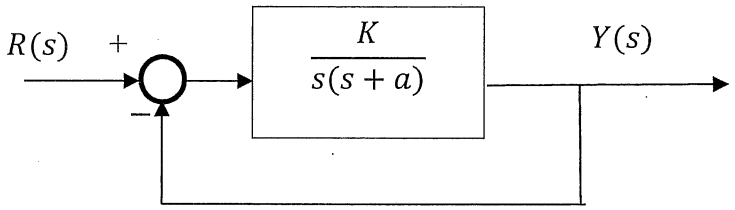

Figure 3: The block diagram considered in problem VII.