

計算題 ※計算題需計算過程，無計算過程者不予計分

1. (35%) An RF (radio frequency) signal with carrier frequency at 1 MHz is expressed as:

$$s(t) = 10 \cos[2\pi f_c t + 10 \sin(2000\pi t) + 10 \sin(8000\pi t)]$$
 - (a) (4%) If this is an FM (frequency modulation) signal and the frequency sensitivity is 10000 Hz/Volt, what is the original message $m(t)$?
 - (b) (4%) What are the corresponding amplitude spectrum, $M(f)$, and the bandwidth of this message?
 - (c) (4%) What is the maximum frequency deviation Δf ?
 - (d) (4%) Find the transmitted bandwidth of this FM signal by Carson's rule.
 - (e) (4%) If the original message $m(t)$ is an analog QAM (quadrature amplitude modulation) modulated signal (i.e., $m(t) = \text{Re}\{\tilde{m}(t)e^{j2\pi f_{IF}t}\}$), what is the intermediate frequency f_{IF} ?
 - (f) (4%) Let $\tilde{m}(t) = I(t) + jQ(t)$, please find the real part $I(t)$ and imaginary part $Q(t)$ of the complex envelope $\tilde{m}(t)$, respectively.
 - (g) (3%) If the original message $m(t)$ in (a) is sampled by an analog-to-digital converter (ADC), what is the Nyquist rate of this signal?
 - (h) (4%) Assume that $m(t)$ in (a) is sampled at 10 kHz and transformed into a pulse-code modulation (PCM) signal by a quantizer. If we limit the transmitted data rate to 60 kbit/s, what is the step size of this quantizer?
 - (i) (4%) Which of the following numbers is the closest one to the maximum signal to noise ratio (in dB unit) of the output? (I) 20 dB (II) 26 dB (III) 30 dB (IV) 36 dB (V) 40 dB. (Note: you need to calculate an approximate value to get full credit.)

2. (15%) An analog signal is sampled, quantized, and encoded into a binary pulse-code modulation (PCM) wave. The number of (quantization) representation levels used is 256. The resulting PCM wave is transmitted over a channel of bandwidth 40 kHz using a binary (two-level) pulse-amplitude modulation (PAM) system with raised-cosine spectrum of rolloff factor 0.25.
 - (a) (4%) Find the bit rate at which information is transmitted through the channel.
 - (b) (7%) Find the rate at which the analog signal is sampled. To avoid aliasing, what is the maximum possible value for the highest frequency component of the analog signal?
 - (c) (4%) Suppose now the PCM wave is transmitted using an octonary (eight-level) PAM system with raised-cosine spectrum of rolloff factor 0.25 under the same transmission bandwidth of 40 kHz. Find the resulting bit rate.

3. (15%) In the on-off keying version of an amplitude-shift keying (ASK) system, symbol 1 is represented by transmitting a sinusoidal carrier:

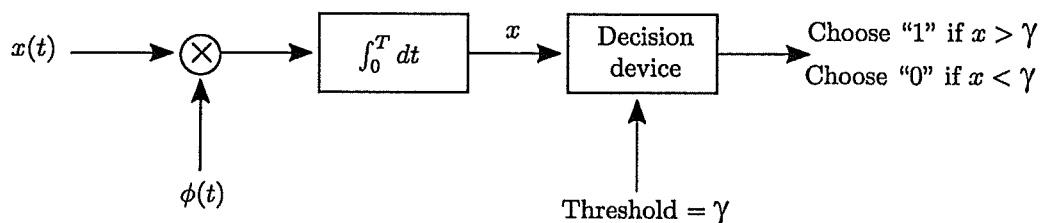
$$s_1(t) = \sqrt{\frac{2E_1}{T}} \cos(2\pi f_c t), \quad 0 \leq t < T$$

where f_c is large compared with $1/T$. Symbol 0 is represented by switching off the carrier:

$$s_0(t) = 0, \quad 0 \leq t < T$$

These ASK signals are transmitted over an additive white Gaussian noise (AWGN) channel with noise of zero mean and power spectral density $N_0/2$.

- (a) (3%) Let $\phi(t)$ be an orthonormal function for this signal set. Find $\phi(t)$.
- (b) (4%) For this ASK, the optimum receiver that minimizes the error probability can be illustrated in the figure shown below, where $x(t)$ is the received signal. Assume equally likely *a priori* probabilities of symbols 1 and 0. Determine the threshold γ .



- (c) (4%) Find the resulting error probability for the optimum receiver with threshold obtained in (b). Express your result in terms of the Q -function given by

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp(-z^2/2) dz.$$

- (d) (4%) Now assume unequal *a priori* probabilities of symbols 1 and 0: $P(s_1) = 1/5$ and $P(s_0) = 4/5$. Determine the corresponding threshold γ for the optimum receiver shown in (b).

4. (20%) Considering noncoherent M -ary frequency-shift keying (FSK) modulation, the individual signal frequencies are separated by the minimum frequency spacing to maintain noncoherently orthogonal and M is a power of 2.

- (a) (4%) Determine the channel bandwidth required to transmit M -ary FSK signals with a bit rate R_b .
- (b) (4%) Assuming that a bandwidth B_1 is required to support a bit rate R_b with $M = M_1$, find the bandwidth B_2 required for supporting the same bit rate with $M = M_2$.
- (c) (6%) Considering an additive white Gaussian noise (AWGN) channel with the received signal power $P = -72$ dBW, bandwidth $B = 64$ kHz, and the noise power spectral density $N_0 = -120$ dBW/Hz, determine E_b/N_0 (the ratio of energy per bit to noise power spectral density) when the maximum information bit rate is achieved.
- (d) (6%) To improve the error performance, a channel code with a code rate $r = 1/3$ is used. If the information bit E_b/N_0 should be improved at least by a factor of 4 in comparison with the E_b/N_0 obtained in (c), determine the maximum achievable information bit rate, the related coded bit rate, and the related value of M .

[Hint: $2 \approx 3$ dB.]

注意：背面有試題

5. (15%) Consider a discrete memoryless Gaussian channel with the channel output $Y = X + N$, where X is an input signal, and N is the additive white Gaussian noise which is independent to X . The channel bandwidth is B and the one-sided noise power spectral density is N_0 .

- (a) (7%) The differential entropy of a continuous random variable X is represented as

$$h(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 [1/f_X(x)] dx,$$

where $f_X(x)$ is the probability density function (pdf) of X . If X is a Gaussian distributed random variable with the pdf

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right],$$

determine the differential entropy of X .

- (b) (3%) Following (a), if $\mu = 0$ and $P = \sigma^2$ is the variance (power) of X , determine the differential entropy of Y .

- (c) (5%) Following (b), according to the fact that the mutual information between X and Y is $I(X;Y) = h(Y) - h(Y|X)$ (where $h(Y|X)$ is the conditional differential entropy of Y given X), determine the information capacity C in bits per channel use.