國立成功大學

114學年度碩士班招生考試試題

編　號：40

系　所：物理學系

科　目：近代物理學

日　期：0211

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. [10] In the Compton scattering, the wavelength difference of the Compton shift is given

$$\Delta\lambda = \lambda - \lambda' = \frac{h}{mc}(1 - \cos\phi),$$

where λ and λ' are the wave length of the incident and scattred photon, respectively, h is the Planck constant, m is the elcetron mass, and ϕ is the scattered angle of λ' to the direction of the incident photon. Show that the fractional loss of energy of a photon during a collision with an electron is given by

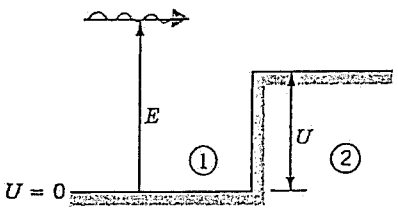
$$\frac{\Delta E}{E} = \frac{hv'}{mc^2}(1 - \cos\phi).$$

Here E is the energy of incdent photon, and v' is the frequency of the scattered photon.

2. [25] Niels Bohr proposed a model for the hydrogen atom based on three key postulates: (1) The electron moves only in certain circular orbits. (2) When the electon transitions from one orbit to another with a lower energy, radiation is emitted, and the radiated frequency is determined by $hv = E_m - E_n$. (3) The orbital angular momentum is quantized by $mvr = n\hbar$, where $\hbar = h/2\pi$. The Coulomb interaction between the electron and proton is given by $k\,e^2/r^2$, (a) [15] Using Bohr's postulate, find the energy spectrun E_n. (b)[10] In the nonrelativistic limit, find the recoil speed of the hydrogen atom when the electron makes a transition from $n = m$ level to $n = m'$.

3. [10] The two protons of mass m_p in the hydrogen molecule are d m apart and rotate about their center of mass. The total angular momentum is quantized in units of $n\hbar$. Find angular velocity ω_n in the nth state.

4. [10] In one-dimensional space, a quantum particle of energy E, which follows the Schrodinger equation, approaches a region, where the potential suddenly rises to U $(E > U)$ as shown in the figure. Find the reflection coefficient that represents the probability of reflection in terms of the wave numbers in the two regions.



5. (a) [10] From a classical viewpoint, assume that an electron is in circular motion around a nucleus. Derive the relation between magetic dipole moment of electron and its orbital angular moemntum. (b) [10] Using the orbital angular momentum in quantum mechanism, find the magitude of the magnetic dipole moemnt of electron in terms of the quantum number of orbital angular momentum (ℓ) and the Bohr magneton, $\mu_B = e\hbar/2m$, where m and e are the mass and charge of the electron, respectively.

6. (a) [10] In classical physics, the addition of velocities in different inertial frames can be given by $v = v' + u$, where v' and v are the velocities of a particle in S' and S frames, respectively, and u is the relative velocity between the frames S and S'. Find the relativistic velocity addition formula in special relativity. (b) [5] Use the result from (a), show that the speed of light is the same in all inertial frames.

7. [10] Before the discovery of neutron, it was hypothesized that the atomic nucleus consisted of protons and electrons. The nucleus has characteristic size of 10^{-15} m, and the average binding energy of particles within the nucleus is less than 10 MeV. Using the uncertainty relation, show that this model is inconsistent with physical observations. (Hint: due to $m_e \ll m_p$, the electron can be regarded as massless particle; $\hbar = 6.58 \times 10^{-16}\ eV \cdot s$; and the speed of light $c = 3 \times 10^8\ m/s$)