

國立成功大學

114學年度碩士班招生考試試題

編 號：176

系 所：企業管理學系

科 目：微積分

日 期：0211

節 次：第 3 節

注 意：1.不可使用計算機
2.請於答案卷(卡)作答，於
試題上作答，不予計分。

Part A: Multiple-choice questions, 32 points, 8 points each. Select all the correct items to get the points.

1. $y = f(x) = x^3 - 3x$, which of the following statements are TRUE?
 - A. When $x = 1$, y reaches the local minimum -2 .
 - B. When $x = -1$, y reaches the local maximum 2 .
 - C. There are three solutions for the equations $f(x) = 1$.
 - D. $f(-x) = f(x)$.
2. $y = f(x) = 2^x$, which of the following statements are TRUE?
 - A. $y = e^{x \cdot \ln 2}$
 - B. $y' = \frac{dy}{dx} = e^x \cdot \ln 2$
 - C. $\int 2^x dx = 2^x \cdot \ln 2$
 - D. $\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{1}{f(k)} = 2$
3. The curve $\Gamma = \{(x, y) \mid x^2 + y^3 + xy = 1\}$, which of the following statements are TRUE?
 - A. Γ has two intersection points with the x -axis.
 - B. Γ has two intersection points with the y -axis.
 - C. $3y + x = 3$ is a tangent line of Γ at $(0, 1)$.
 - D. $2y - x = 3$ is a tangent line of Γ at $(-1, 1)$.
4. $f(x) = \sqrt{4 - x^2}$, which of the following statements are TRUE?
 - A. $f'(x) = \frac{1}{\sqrt{1-x^2}}$
 - B. $f''(x) = -4 \cdot (4 - x^2)^{-\frac{3}{2}}$
 - C. $\int_{-1}^1 f(x) dx = \pi$
 - D. $\int_{-1}^1 f'(x) dx = 0$

Part B: Fill in the blanks, 28 points, 7 points each. Derivations are not required.

5. For a polynomial $f(x)$, if $f(2) = 3$ and $f'(2) = 4$. Use the linear approximation of $f(x)$ at $x = 2$ to estimate $f(2.01) = \underline{\hspace{1cm}} (5) \underline{\hspace{1cm}}$.
6. The area of the region Ω which is enclosed between $y = f(x) = x(4 - x)$ and $y = g(x) = x^2$ is $\underline{\hspace{1cm}} (6) \underline{\hspace{1cm}}$.
7. Continuing from the previous question 6, the region Ω is revolved around the x -axis to form a solid of revolution. The volume of this solid is $\underline{\hspace{1cm}} (7) \underline{\hspace{1cm}}$.
8. The Taylor polynomial for $y = f(x) = \ln(1 + x^2)$ up to the x^6 term is $\underline{\hspace{1cm}} (8) \underline{\hspace{1cm}}$.

Part C: Answer the following questions, 40 points, 10 points each. Derivation is required. Simplify your answer as possible as you can.

9. Compute the indefinite integral $\int \frac{1}{x^2(x+1)} dx$.

10. Compute the double integral $\int_0^2 \int_0^x xy \, dy \, dx$.

11. Solve the differential equation: $\frac{dy}{dx} = y - xy$ with the initial condition: $y = 1$ when $x = 0$.

12. A company makes a product and the cost function $C(q)$ and demand function $D(x)$ are given by $C(q) = q^2 + 4q + 200$ and $D(x) = 120 - x$, respectively, where q represents the quantity produced, and x represents the price per product. The company's challenge is to determine the optimal product pricing to maximize profits. The profit function is defined as $P(x) = x \cdot D(x) - C(D(x))$. Please derive the profit function, find the critical points of the profit function and determine whether these critical points are maximum or minimum and identify the optimal price.