國立成功大學

114學年度碩士班招生考試試題

編　號：211

系　所：環境醫學研究所

科　目：微積分

日　期：0211

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (24%) Evaluate the following limits:

   (a) $\lim_{x\to 0}(1+\sin 2x)^{\frac{1}{3x}}$.

   (b) $\lim_{x\to 0}\frac{\tan^{-1}(ax)}{\tan^{-1}(bx)}$, where a and b are nonzero constants.

   (c) $\lim_{x\to 0+}\frac{\sqrt{x-\ln(1+x)}}{x}$.

   (d) $\lim_{x\to 0+}\frac{\int_0^{x^2} e^t\sqrt{t}\sin\sqrt{t}\,dt+x\cos x-x}{x^3}$.

2. (10%) Find $f'(\frac{\pi}{4})$ if $f(x)=e^{g(x)}$ and $g(x)=\int_1^{\tan x}\sqrt{1+t^3}\,dt$.

3. (10 %) Let $F(x)=\int_0^x\left(\int_0^{u^3} f(t)\,dt\right)du$ and $G(x)=\int_0^{x^3} f(u)(x-\sqrt[3]{u})\,du$, $x\geq 0$. Show that $F(x)=G(x)$ for $x\geq 0$.

4. (10%) A number a is called a fixed point of a function $f(x)$ if $f(a)=a$. Prove that if $f(x)$ is differentiable and $f'(x)\neq 1$ for all real number x, then f has at most one fixed point.

5. (10%) Evaluate the iterated integral $\int_0^1\int_y^1\tan(x^2)\,dxdy$.

6. (15%) Suppose that $z=f(x,y)$ is a smooth function and let $x=uv$, and $y=v-u$. Express $\frac{\partial^2 z}{\partial u\partial v}$ in terms of $x, y, f_x, f_y, f_{xx}, f_{xy}$, and f_{yy}.

7. (10%) Find the equation of the tangent line to the curve satisfying $x^{\frac{2}{3}}+y^{\frac{2}{3}}+y=6$ at $(8,1)$.

8. Let $f(x)$ be a continuous function and $\int_0^x f(u)\,du=-2+x^2+x\sin 2x+c\cos 2x$, $x\geq 0$.

   (a) (3%) Find the value c.

   (b) (8%) Find $f'(\frac{\pi}{4})$.