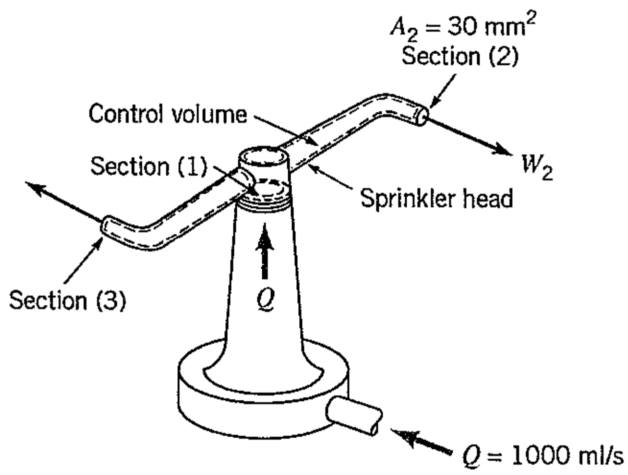
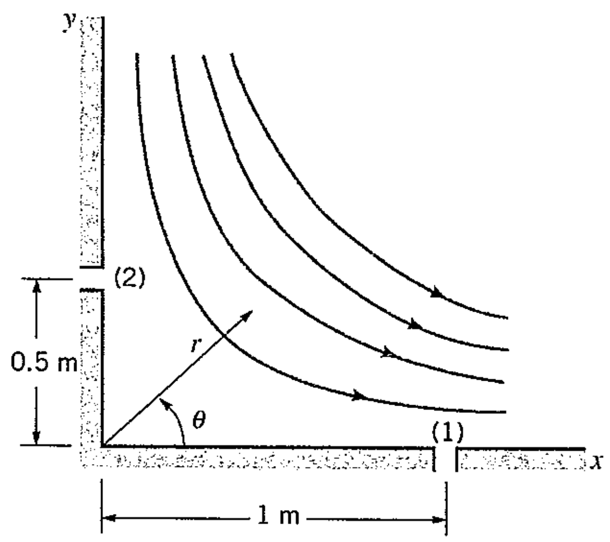
1. Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in the following figure. The exit area of each of the two nozzles is 30 mm^2. Determine the average speed of the water (m/s) leaving the nozzle, relative to the nozzle, if
(a) (15%) the rotary sprinkler head is stationary,
(b) (5%) the sprinkler head rotates at 600 rpm, and
(c) (5%) the sprinkler head accelerates from 0 to 600 rpm.



2. The two-dimensional flow of a non-viscous, incompressible fluid in the vicinity of the 90^0 corner as shown in the following figure is described by the stream function $\psi = 2r^2 \sin 2\theta$, where ψ has units of m^2/s when r is in meters. Assume the fluid density is 10^3 kg/m^3 and the $x-y$ plane is horizontal, that is, there is no difference in elevation between points (1) and (2).
(a) (15%) Determine the velocity potential function ϕ.
(b) (10%) If the pressure at point (1) on the wall is 30 kPa, what is the pressure at point (2)?



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3. Consider an incompressible Newtonian fluid with dynamic viscosity μ and density ρ, which is flowing steadily through a horizontal long circular straight pipe with diameter D at an average speed V. We can express the pressure drop per unit axial length, $\Delta p/\Delta x$, as $\Delta p/\Delta x = f(D,\rho,\mu,V)$.

(a) (10%) Show that $\Delta p/\Delta x$ can be expressed in dimensionless form as $\frac{\Delta p/\Delta x}{\rho V^2/D} = f(\mathrm{Re})$, where $\mathrm{Re} = \frac{\rho V D}{\mu}$ is the Reynolds number, through the use of dimensional analysis and Buckingham pi theorem.

(b) (15%) Considered the flow as fully-developed and axisymmetric. The velocity field can be expressed as $\mathbf{u} = u(r)\mathbf{e}_x$ in a cylindrical coordinates system, (r, θ, x), fixed with respect to the pipe, with the coordinate center at the central point of the inlet plane, and x-axis along the pipe axis. According to the momentum balance, we can write: $\mu\left(\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr}\right) = \frac{dp}{dx} = \text{constant} = \frac{\Delta p}{\Delta x}$.

Solve the above equation for $u(r)$ subject to $u(D/2) = 0$, and derive that $\frac{\Delta p/\Delta x}{\rho V^2/D} = -\frac{32}{\mathrm{Re}}$.

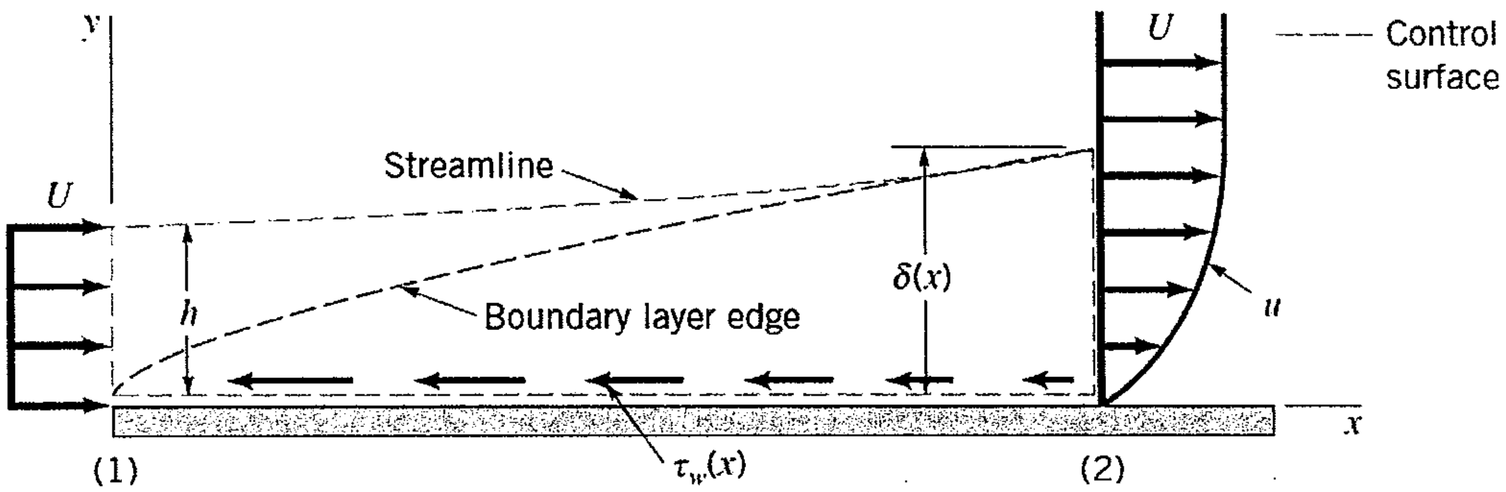
4. The following figure shows the steady incompressible boundary layer flow over a flat plate.

(a) (15%) Derive the momentum integral equation using the conservation of linear momentum for the flow through the control volume indicated. The result should be

$$\rho U^2 \frac{d\theta}{dx} = \tau_w,$$

where $\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$ and $\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$ are the momentum thickness and the shear stress at wall, respectively.

(b) (10%) Determine the boundary layer thickness and τ_w, assuming a linear velocity profile across the boundary layer, i.e., $u = U\frac{y}{\delta}$ for $0 \le y \le \delta$ and $u = U$ for $y > \delta$, with $\delta = \delta(x)$ the boundary layer thickness.



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