1. (10%) Find the local maxima and minima of the function: $f(x,y) = \frac{1}{3}x^3 + xy^2 - 2xy + 2$.

2. (5%) Assume the polynomial function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ satisfies $f(-2) = 112$, $f(-1) = -21$, $f(0) = 3$, $f(1) = -18$, and $f(2) = 120$. Find a, b, c, d, and e.

3. (15%) Let A be a 2×2 matrix with eigenvalues of $\lambda_1 = 3$ and $\lambda_2 = -1$ and corresponding eigenvectors of $v_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, respectively.
   a. Find A. (3%)
   b. Find the eigenvalues for $A + I$. (2%)
   c. Calculate $(A + I)^{1000}$. (10%)

4. (10%) Assume the matrix $\begin{bmatrix} 1 & 2 & 1 & 1 & b \\ 2 & 5 & 3 & a & 0 \\ 1 & 0 & 6 & 4 & c \end{bmatrix}$ can be transformed to the reduced row echelon form of $\begin{bmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & d & -1 \\ 0 & 0 & 1 & 1 & e \end{bmatrix}$. Find a, b, c, d, and e.

5. (30%) Let $y = y(x)$ be a function of the variable x.
   a. Solve $y'' + 4y' - 2y = 0$. (10%)
   b. Solve $y'' + 4y' + 4y = 0$. (10%)
   c. Solve $y'' + 2y' + 6y = 0$. (10%)

6. (5%) Let a class C^1 vector field $\boldsymbol{v} = P(x,y)\hat{\imath} + Q(x,y)\hat{\jmath}$ be defined in a region $\mathcal{R}$ in two-dimensional space. Let $\mathcal{S}$ be a region within $\mathcal{R}$, and let the edge of $\mathcal{S}$ be a piecewise smooth simple closed curve $\mathcal{C}$, oriented counterclockwise. Given $\int_{\mathcal{S}} \hat{\boldsymbol{n}} \cdot \nabla \times \boldsymbol{v}\, dA = -\frac{4}{15}$, where $\hat{\boldsymbol{n}} = \hat{\boldsymbol{k}}$ and dA is an infinitesimal area element on $\mathcal{S}$, while $\hat{\imath}$, $\hat{\jmath}$ and $\hat{\boldsymbol{k}}$ are unit vectors of the cartesian coordinate system such that $\hat{\imath} \times \hat{\jmath} = \hat{\boldsymbol{k}}$. Please determine $\oint_{\mathcal{C}} P dx + Q dy$, and clearly show the process that you use to determine it.

7. (15%) Let $\boldsymbol{v}$ be a class C^1 vector field in a simply connected domain $\mathcal{D}$, and $\mathcal{C}$ be a piecewise smooth simple closed path lying within $\mathcal{D}$. Knowing that there exists a class C^2 scalar function Φ in $\mathcal{D}$ such that $\boldsymbol{v} = \nabla\Phi$ throughout $\mathcal{D}$. Please answer the following questions.
   (a) (5%) Determine $\nabla \times \boldsymbol{v}$.
   (b) (5%) Determine $\int_{\mathcal{C}} \boldsymbol{v} \cdot d\boldsymbol{R}$, where $d\boldsymbol{R}$ is an infinitesimal line element on $\mathcal{C}$.
   (c) (5%) If $\boldsymbol{v}$ is a fluid velocity field, state the condition of the angular velocity of the fluid.

8. (10%) Solving the following partial differential equation using the Laplace transform method (or another method if it may work).

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad (0 < x < 1, 0 < t < \infty)$$

$$u(0,t) = 1, \quad u(1,t) = 1, \quad (0 < t < \infty)$$

$$u(x,0) = 1 + \sin \pi x. \qquad (0 < x < 1)$$

試題隨卷繳回