1. (25 points) A prismatic bar with the length $L = 1$ m and the area of cross section $A = 1 \times 10^{-4}$ m^2 is made by a linearly elastic material ($\sigma = E\epsilon$ and $E = 1 \times 10^2$ GPa) and subjected to the distributed force $p(x) = x$ kN/m as shown in Figure 1.

   (a) (15 points) Please calculate the axial displacement field $u(x)$, the normal strain field $\epsilon(x)$, the normal stress field $\sigma(x)$ and the axial force field $N(x)$ of the bar.

   (b) (5 points) Please determine the maximum shear stresses $\tau_{\max}(x)$ at every point in the bar and indicate the inclined angle $\alpha(x)$ of the planes on which they act.

   (c) (5 points) If the right end of the bar becomes a fixed end as shown in Figure 2, please calculate the axial displacement field $u(x)$ of this fixed-fixed bar.

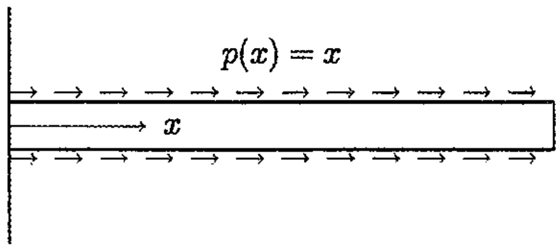

Figure 1: Schematic diagram of a fixed-free bar subjected a distributed load.

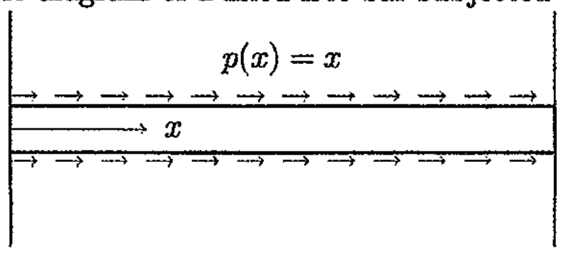

Figure 2: Schematic diagram of a fixed-fixed bar subjected a distributed load.

2. (20 points) A circular tube with the length $L = 1$ m and the outer and the inner radii $R_{\text{out}}(x) = (2-x)^{1/4}$ m and $R_{\text{in}}(x) = 0.8 \times (2-x)^{1/4}$ m is made by a linearly elastic material ($\tau = G\gamma$ and $G = 5 \times 10^2$ GPa) and subjected to the concentrated torque $T_c = 5$ kN·m at the end as shown in Figure 3. Please calculate the twist angle field $\phi(x)$, the shear strain field $\gamma(r,x)$, the shear stress field $\tau(r,x)$, the torsional moment field $T(x)$, and the stored energy per unit volume $U_v(r,x)$ of the tube where r is the radial distance and $R_{\text{in}}(x) \leq r \leq R_{\text{out}}(x)$.

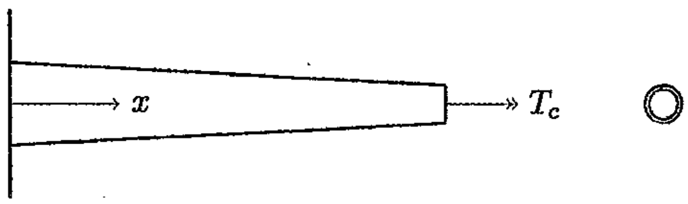

Figure 3: Schematic diagram of a circular tube subjected a torque.

3. (30 points) A simply supported beam with the length L and the moment of inertia I is made by a linearly elastic material ($\sigma = E\epsilon$) and subjected to the distributed vertical load $q(x)$ as shown in Figure 4.

   (a) (15 points) Please determine the shear forces $V(0)$ and $V(L)$, the bending moments $M(0)$ and $M(L)$, the curvatures $\kappa(0)$ and $\kappa(L)$, the rotation angles $\theta(0)$ and $\theta(L)$, and the deflections $w(0)$ and $w(L)$ of the beam in terms of the distributed load $q(x)$ (with detailed derivations).

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(b) (10 points) If $q(x) = q_0$ where q_o is a constant, please derive the formulation of the shear force $V(x)$, the bending moment $M(x)$, the curvature $\kappa(x)$, the rotation angle $\theta(x)$, and the deflection $w(x)$ of the beam in terms of q_0 (with detailed derivations).

(c) (5 points) If $q_0 = 36$ kN/m, the length of the beam $L = 20$ m and the cross section is rectangular with width $b = 0.2$ m and height $h = 0.6$ m, please plot the stress element of principal stresses and determinate the maximum shear stress on the line ($x = 15$ m, $z = 0.1$ m).

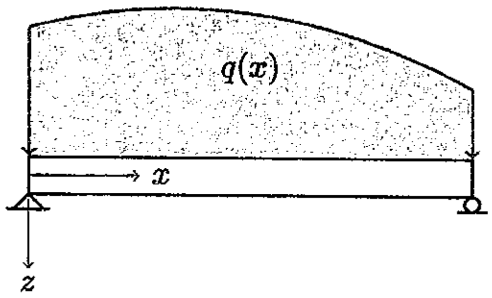

Figure 4: Schematic diagram of a simply support beam subjected a distributed load.

4. (25 points) A prismatic column as shown in Figure 5 has length L and circular cross-section with radius R m. The column is made by a linearly elastic material ($\sigma = E\epsilon$) with pinned ends and is subjected to a *compressive* load P. The displacement of the column along the z-direction is denoted by $w(x)$.

(a) (5 points) Plot the free body diagram to formulate the relationships between the shear force $V(x)$ and the bending moment $M(x)$ of the column.

(b) (10 points) Please derive the buckling equation (differential equation) if Young's modulus is a function of x, *i.e.* $E(x)$ and formulate the general solution of the buckling equation if Young's modulus E is constant.

(c) (10 points) Please derive the buckling load P_{cr} in terms of E, L and R and plot the buckled mode shape (with detailed solution process).

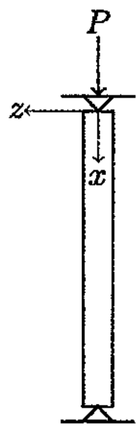

Figure 5: Schematic diagram of a column with pinned-pinned supports.

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