- This exam is closed-book and closed-notes, and calculators are not permitted.
- Please present your solutions with detailed steps. If any shortcuts are used, provide a written explanation to justify them.

1. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) **(5%)** Find the determinant of $5A^7$

(b) **(5%)** Find the eigenvalues of $5A^7$

(c) **(12%)** Diagonalize A

(d) **(8%)** Calculate A^{2025} using diagonalization

2. **(15%)** Find the Fourier series representation of $f(x) = \sin(x)\cos(x)$ in the interval $(-\pi,\ \pi)$.

3. **(15%)** Solve the following ordinary differential equation, and express the answer in explicit form, i.e., $y = f(x)$.

$$\frac{d}{dx}[xy] = 2y - xe^{y/x}\ ,\quad y(e^2) = 0\ ,\quad x > e$$

4. **(20%)** Solve the following partial differential equation, and express the answer in explicit form, i.e., $u = f(x,y)$.

$$3xu_x - u_y = 2u - x\ ,\quad u(x,0) = 3x$$

5. **(20%)** Solve the following ordinary differential equation using the Laplace transform.

$$y'' - 3y' + 2y = e^{3t}\ ,\quad y(0) = 1\ ,\ y'(0) = 0$$

Recall that $\mathrm{L}\{f'(t)\} = s\mathrm{L}\{f(t)\} - f(0)$　and　$\mathrm{L}\{e^{at}\} = \dfrac{1}{s-a}$

試題隨卷繳回