考試科目	統計學	系所別	財務管理學系	考試時間	2 月 12 日(三) 第 四 節

一、填充題或簡答題 (83%)

註：若該小題沒有標註為簡答題，皆為填充題。填充題只需寫出答案，不需計算過程。
註：請按照題號作答，否則不予計分。

1. To investigate the relationship between the number of years of experience (x) and the annual sales of a salesperson (Y), a sales manager randomly selected 10 salespersons to collect data. The 10 observations of (x, y) are denoted by (x_i, y_i), $i = 1, \ldots, 10$. The data are given in the following table. Consider a simple linear regression model

$$Y = \beta_0 + \beta_1 x + \varepsilon.$$

id	1	2	3	4	5	6	7	8	9	10
x	2	5	5	4	7	9	14	11	11	12
Y	83	95	105	100	106	114	139	122	126	120

Note: It is known that $\sum_{i=1}^{10} x_i = 80$, $\sum_{i=1}^{10} y_i = 1110$, $\sum_{i=1}^{10} x_i y_i = 9448$, $\sum_{i=1}^{10} x_i^2 = 782$, and $\sum_{i=1}^{10} y_i^2 = 125652$.

(a) (5%，簡答題) Why do we have to incorporate the error term, ε, in the model?
(b) (5%) Find the least squares estimates of β_0 and β_1.
(c) (5%) Given that the residuals are $-6, -5, -4, -3, 1, 1, 1, 4, 4, 7$ (not ordered in their id), calculate the sum of squares due to error.
(d) (5%) Compute the coefficient of determination. Comment on the goodness of fit. (簡答)

2. The *LifeCycleSavings* dataset from Belsley, Kuh, and Welsch (1980) is adopted to investigate how the savings ratio is explained some independent variables, including the percentage of population less than 15 years old (pop15; X_1), the percentage of the population over 75 years old (pop75; X_2), per-capita disposable income (dpi; X_3), and the percentage rate of change in per-capita disposable income (ddpi; X_4). There are 50 observations in the dataset. The regression model is

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon.$$

Partial of the outputs are given on Page 2.

(a) (5%) Find the (D1) and (D2) in the ANOVA table.
(b) (5%) Find the (D4) in the ANOVA table.
(c) (5%) What is the interpretation of $\hat{\beta}_1$?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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(Cont. 2)

Source of Variation	Sum of Squares	Degree of Freedom	Mean Square	F
Regression	(D1)	(D2)	(D3)	(D4)
Error	650.71	(D5)	(D6)	
Total	983.63	(D7)		

Predictor	Coef	SE Coef
Constant	28.57	7.35
pop15	−0.46	0.14
pop75	−1.69	1.08
dpi	−0.0003	0.0009
ddpi	0.41	0.20

3. (5%) Consider the hypothesis test: H_0: $\mu \leq 100$ v.s. H_a: $\mu > 100$, where μ is the population mean. A sample of 100 individuals is drawn and surveyed. The population standard deviation is known to be 5. Using a 0.02 level of significance, what is the power when the actual population mean is 102 ?

4. A survey in 2024 showed that 70% of all individuals ages 66 to 75 are well prepared financially for retirement. Many financial planners express their concern that a smaller percentage of individuals in the 56 to 65 age group are well prepared. Denote p be the population percentage of individuals in the 56 to 65 age group being well prepared financially for retirement. Conduct a hypothesis testing H_0: $p \geq 0.7$ v.s. H_a: $p < 0.7$, and α is set to be 0.01.
   (a) (5%) In a random sample of 200 individuals from the 56 to 65 age group, 65 were not well prepared financially for retirement. Compute the smallest level of significance for which we could reject the null hypothesis.
   (b) (5%，簡答題) Explain why should we compute the quantity in (a) when we mention "smallest level of significance for which we could reject the null hypothesis."

5. The website for a certain hotel in Taipei gets, on average, six visitors per hour. Suppose the number of website visitors per minute follows a Poisson probability distribution.
   (a) (5%) What is the probability that no one will access the website in a 12-minute period?
   (b) (5%) What is the probability that the time between website visitors will be between 6 minutes and 12 minutes?

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6. There are three main supermarket chains (supermarket A, B, C) in the Denver area, and each of them claims that their chain has the lowest overall grocery prices. Students taking the statistics course try to conduct a survey to determine whether the mean price of groceries are the same across the three supermarket chains. Students randomly selected six grocery items. They visited the three supermarkets on the same day, and each of the six items was purchased once in each of the three supermarkets. The data were recorded in the following table.

Assume that the error term of one treatment follows a Normal distribution, and these Normal distributions have the same variance. Assume further that responses are independent. Use a 0.05 level of significance.

Item	supermarket A	supermarket B	supermarket C
1	11	10	12
2	16	19	22
3	22	20	24
4	39	44	43
5	36	32	37
6	56	55	60
sample mean	30	30	33
sample variance	282.8	289.2	297.6

(a) (7%) Compute the Sum of Squares for Treatment.

(b) (8%) What is the value of the test statistics for testing whether the mean price of products are the same across the three supermarket chains?

(c) (8%，簡答題) Suppose that we further find out the critical value of this testing, and the result is to reject the null hypothesis. What is your interpretation about this result? What is your plan for the next step of analysis (what is the tool or method you are going to use)?

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二、計算題或簡答題 (17%)

註：請寫出計算過程，否則不予計分。請清楚標示最後的答案。

7. Let μ denote the population mean. To make inference about μ, we generally use the sample mean to estimate μ. Based on observations $x_1, x_2, \ldots, x_n$, we are able to compute sample mean $\bar{x}_n$.

(a) (2%) Explain why it is impossible to compute the difference between the sample mean and the population mean, that is, $\bar{x}_n - \mu$.

(b) (5%) Provide an alternative of (a) to quantify the "difference" between μ and point estimation, and this alternative is computable. Specify every components in your equation, and explain why this alternative is computable.

8. Suppose that the population standard deviation is known. To make inference about the population mean μ, a simple random sample of 100 individuals is surveyed, and the resulting 99% interval estimate for μ is [127.94, 132.06].

Note: $z_{0.005} = 2.575$.

(a) (5%) Please compute the 95% confidence interval of μ based on the same data.

(b) (5%) Give your interpretation for the numerical interval [127.94, 132.06].

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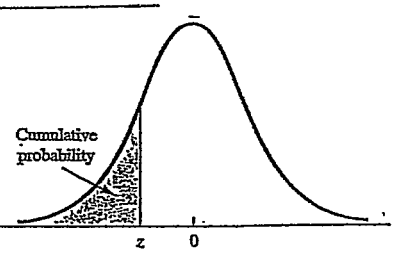
Table of the standard Normal Distribution (its cumulative probabilities)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。