國立政治大學 114 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	統計學	系所別	經濟學系	考試時間	2 月 11 日（二）第四節

Instructions:

- *Please label the question numbers and answer them in numerical order.*
- *You must show your work and write complete answers. Answers with no explanations or derivations, or incorrect ones, will receive a score of zero.*

1. (25%) Let $X_1, \ldots, X_n$ be a random sample from the $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $0 < \sigma < \infty$. Consider the estimation of σ^2 using the squared error loss. Let S^2 be the sample variance of $X_1, \ldots, X_n$.

   (a) (4%) Find $E[S^2]$ and $\text{Var}[S^2]$.

   (b) (5%) Find $E[S^2]$ if the population distribution is unknown with mean μ and variance σ^2.

   (c) (8%) Define a class of estimators of the form cS^2 with a nonrandom c. Compute the risk function in this class and find the best estimator that minimizes the risk.

   (d) (8%) Suppose that $\sqrt{n}(S^2 - \sigma^2)$ converges to $N(0, V)$ in distribution as $n \to \infty$. Using the Delta method find the limiting distribution of $\sqrt{n}(S - \sigma)$, where S is the sample standard deviation.

2. (25%) Suppose that $X_1, \ldots, X_n$ is a random sample from $U[0, \theta]$, where U denotes a uniform distribution.

   (a) (5%) Find the maximum likelihood estimator of θ, denoted as $\hat{\theta}$.

   (b) (5%) Find the sampling distribution of $\hat{\theta}$. (Provide the probability density function.)

   (c) (10%) Find $E[\hat{\theta}]$ and $\text{Var}[\hat{\theta}]$.

   (d) (5%) Is $\hat{\theta}$ a consistent estimator for θ? (Use the definition of convergence in probability to demonstrate (in)consistency.)

3. (16%) Suppose that the true process is $Y = X + X^2$ with $X \sim U[0, 1]$. Consider the model $Y = \beta_0 + \beta_1 X + u$ with the error u satisfying $E[u] = E[Xu] = 0$.

   (a) (8%) Use the model restrictions directly to calculate β_0 and β_1.

   (b) (2%) Find $E[u|X]$.

   (c) (3%) Denote $\hat{\beta}_1$ the OLS estimator of β_1. Would you agree that $\hat{\beta}_1$ consistently estimates the average marginal effect of X on Y? Why or why not?

   (d) (3%) Assume the data are divided into two groups, each following $Y = X + X^2$ but with a distinct marginal distribution of X, resulting in their respective (β_0, β_1) calculated as in (a). Would you agree that the effect of X on Y differs across groups? Why or why not?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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4. (34%) Using a sample of 532 workers, a researcher is interested in whether there is a tradeoff between the time spent sleeping per week and the time spent in paid work. The regression results are presented below, with standard errors in parentheses and SER denoting the standard error of the regression.

   The variable *sleep* is total minutes per week spent sleeping at night; *work* is total weekly minutes spent working; *educ* is years of schooling; $male = 1$ for men and $male = 0$ for women; $D_{age\geq 40} = 1$ for a worker aged 40 or older and $D_{age\geq 40} = 0$ otherwise; and *hrwage* is hourly wages in dollars.

	Dependent Variable: *sleep*			
	(1)	(2)	(3)	(4)
constant	3301.8	3578.73	3719.58	3725.53
	(36.5)	(44.76)	(95.51)	(97.48)
work		−0.16	−0.16	−0.16
		(0.02)	(0.02)	(0.02)
educ			−11.74	−11.30
			(6.86)	(6.86)
male	−115.8	45.90	45.35	50.15
	(48.3)	(38.86)	(42.42)	(42.64)
$D_{age\geq 40}$	−2.9			
	(56.2)			
$male \times D_{age\geq 40}$	110.2			
	(76.4)			
ln(*hrwage*)			8.63	
			(3.30)	
hrwage				−1.12
				(1.21)
$hrwage^2$				0.07
				(0.05)
Observations	532	532	532	532
SER	429	408	400	400
R^2	0.015	0.107	0.122	0.122

(a) (4%) The sample can be divided into four groups based on *male* and $D_{agd\geq 40}$. Calculate the sample mean of sleep time per week for each group based on coefficient estimates in Column (1).

(b) (4%) If *sleep* is measured in hours in Column (1), what are the coefficient estimate and standard error for *male*, as well as the SER and R^2?

(c) (3%) A student is unhappy with model (2) as "a female dummy is omitted from the model." Comment on this criticism.

(d) (3%) Given Column (2) and the estimated regression $\widehat{male} = 0.086 + 0.0002 \times work$, determine the slope coefficient estimate in the simple regression of *sleep* on *work*.

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4. (continued)

(e) (3%) Interpret the coefficient estimate of 8.63 for ln(*hrwage*) in Column (3).

(f) (3%) Following (e), does the partial effect of hourly wages on mean sleep time decrease, remain constant, or increase with each one-dollar rise? Explain your reasoning.

(g) (3%) Based on Column (3), all other factors being equal, is there a statistically significant tradeoff between working and sleeping? How strong is the evidence? (The 0.95, 0.975, 0.99, 0.995, and 0.9995 quantiles of $N(0,1)$ are 1.64, 1.96, 2.33, 2.58, and 3.29.)

(h) (3%) Based on Column (4), determine the hourly wage level at which weekly sleep time starts increasing as hourly wages rise.

(i) (4%) Adjust model specification (2) to test whether the tradeoff between sleep and work time varies by gender. State the relevant null and alternative hypotheses. Provide an appropriate test statistic (with its explicit form) and its asymptotic null distribution.

(j) (4%) Based on Column (4), a researcher wants to test the hypothesis that, holding other factors fixed, hourly wages have no effect on sleeping. State the relevant null and alternative hypotheses. Provide an appropriate test statistic (with its explicit form) and its asymptotic null distribution.

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