考試科目	微積分	系所別	應用數學系	考試時間	2 月 12 日(三) 第三節

Show all your work to earn the credits.

1. (8 points) Evaluate the integral: $\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1} dydx$

2. (8 points) Evaluate the double integral:

$$\iint_D y + \sqrt{x^2+y^2} dA$$

where $D = \{(x,y) : x \leq 0, y \geq 0, x^2+y^2 \leq 9\}$.

3. (8 points) Evaluate the integral: $\int \frac{\tan^{-1} x}{1+\frac{1}{x^2}} dx$.

4. Consider the function

$$f(x,y) = \begin{cases} \frac{2x^2y+y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) (5 points) Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$.

(b) (5 points) Find $\frac{\partial f}{\partial x}(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ for all $(x,y) \in \mathbb{R}^2$.

(c) (5 points) Is $f(x,y)$ differentiable at $(0,0)$? Give a valid reason for your answer.

5. (10 points) Determine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!+1)^2}{[(n+1)!]^2}$$

and give a valid reason for your answer.

6. (10 points) Consider the surface $f(x,y) = x^2y+5$. Find the absolute maximum and absolute minimum of $f(x,y)$ over the region $R = \{(x,y) : x \leq 0, y \geq 0, x^2+y^2 \leq 9\}$.

7. (10 points) Find the first four non-zero terms of the Taylor series about 0 for the function

$$f(x) = xe^{x^2} - \frac{1}{4+x^2}.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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8. Let C be the piecewise smooth curve given by

$$\vec{r}(t) = \begin{cases} \langle e^{t^2} - 1, e^{t^4} - 1 \rangle, & \text{for } 0 \le t \le 2 \\ \langle e^{2t} - 1, e^{8t} - 1 \rangle, & \text{for } 2 < t \le 3 \end{cases}$$

and let $\vec{F}(x, y) = \langle ye^x, e^x + 3y^2 \rangle$ be a vector field.

   (a) (8 points) Is $\vec{F}(x, y)$ conservative? If so, find a potential function for $\vec{F}$. If not, explain why not.

   (b) (8 points) Calculate the integral $\int_C \vec{F} \cdot d\vec{r}$.

9. (5 points) Prove that the two surfaces

$$S_1 : x^2 + y^2 + z^2 = 5 \quad \text{and} \quad S_2 : x^2 = 4y^2 + 4z^2$$

are perpendicular (orthogonal) to each other at all points of intersection.

10. Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence of real numbers. For each $n \in \mathbb{N}$, define

$$M_n = \sup\{a_n, a_{n+1}, a_{n+2}, \cdots\} \quad \text{and} \quad m_n = \inf\{a_n, a_{n+1}, a_{n+2}, \cdots\}.$$

   (a) (5 points) Show that $\lim_{n\to\infty} M_n$ and $\lim_{n\to\infty} m_n$ exist.

   (b) (5 points) Show that, if $\lim_{n\to\infty} M_n = \lim_{n\to\infty} m_n$, then $\{a_n\}_{n=1}^{\infty}$ converges.

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