逢甲大學113學年度碩士班考試入學試題

編號：06　　科目代碼：104

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	90分鐘

※請務必在答案卷作答區內作答。　　共 2 頁 第 1 頁

1. (20%) Find the Limits
   a. (5%) $\lim_{x\to 3}\frac{x^2-9}{\sqrt{x^2+7}-4}$.
   b. (5%) $\lim_{x\to 0}\frac{\sqrt[p]{1+x}-1}{x}, p>0$.
   c. (5%) $\lim_{x\to\infty}\frac{x^{-1}+x^{-4}}{x^{-2}+x^{-3}}$.
   d. (5%) $\lim_{x\to\infty}\sqrt{x^2+x}-\sqrt{x^2-x}$.

2. (10%) Find Derivative dy/dx
   a. (5%) $y=\left(\frac{x}{5}+\frac{1}{5x}\right)^5$, what is $y'(1)$?
   b. (5%) $y^2=\sqrt{\frac{1+x}{1-x}}$, what is $y'(0)$?

3. (10%) Application of Derivative
   a. (5%) Suppose that the first derivative of $y=f(x)$ is

$$y'=6(x+1)(x-2)^2.$$

   At what points, if any, does the graph of f have a local maximum, local minimum, or point of inflection?
   b. (5%) For what value or values of the constant k will the curve $y=x^3+kx^2+3x-4$ have one horizontal tangent?

4. (10%) Cost and Avenue in Economics

   Suppose that $c(x)=x^3-6x^2+15x$ is the cost function of producing the x (in thousand of units) items. And define the marginal cost $m(x)=c'(x)$ and the average cost $a(x)=c(x)/x$. It is well known that the production level (if any) at which the average cost is *smallest* is a level at which the average cost equals the marginal cost.
   a. (5%) Find the two production levels x_1^* and x_2^* that possibly minimizes average cost.
   b. (5%) Which level minimizes the average cost and why?

5. (10%)

Let $f(x)$ be a function satisfying $\int_0^\infty f(x)\mathrm{d}x = 1$ and let $F(y) = \int_0^y f(x)dx$.

Show that

$$\int_0^\infty [1 - F(y)]\, dy = \int_0^\infty x f(x) dx.$$

6. (30%)

Given two numbers $\mu \in \mathbb{R}$ and $\sigma^2 > 0$, we define the functions

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$$

and

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy.$$

If we know that $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(1) = 1$, and $\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$. Please evaluate the following integral with detailed derivations.

a. (10%)

$$\int_{-\infty}^\infty f(y) dy.$$

b. (10%)

$$\int_{-\infty}^\infty y f(y) dy.$$

c. (10%)

$$\int_{-\infty}^\infty y^2 f(y) dy.$$

7. (10%)

Consider the Euler integral of the first kind as

$$B(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx, \quad \text{for } a, b > 0.$$

Find the value of $B(1/2,1/2)$.