1. Solve the initial value problem of the following differential equations.

(1). $y' = -(x^2+2)y + e^{x^3}y^4, y(0) = 2$ (10%)

(2). $\cosh(x-y) + x\sinh(x-y) - x\sinh(x-y)y' = 0; y(4) = 4$ (10%)

(3). $\begin{cases} y'_1 = -3y_1 - 4y_2 + 2e^t \\ y'_2 = 5y_1 + 6y_2 - e^t \end{cases}$ , $y_1(0) = 1, y_2(0) = 3$ (10%)

(4). $x^3 y''' - x^2 y'' - 7xy' + 16y = 18x^3 \ln|x|, \quad y(1) = 1\ y'(1) = 2, y''(1) = 3$ (10%)

2. Find the solutions of the following differential equations by using Laplace transforms.

(1). $y'' + 5y' + 6y = e^{-t}[5u(t-1) - 2u(t-2)] + 4\delta(t-3), y(0) = 0, y'(0) = 0$ (10%)

(2). $y(t) + 2\int_0^t y(\tau)\cos(t-\tau)d\tau = \cos t + t$ (10%)

(3). $y'' - 6y' + 5y = \begin{cases} 150, & 0 < t < 2 \\ 0, & 2 < t < 4 \end{cases}, T = 4(cycle)(periodic\ function), \quad y(0) = 1, y'(0) = 2$ (10%)

4. Calculate the following integral of complex functions using Residue theorem

(1). $\int_0^\pi \dfrac{d\theta}{16 - 4\sin\theta}$ (7%)

(2). $pr.v.\int_{-\infty}^\infty \dfrac{x^2}{x^4 - 1}dx$ (8%)

5. The matrix $B$ is as below.

$$B = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

(1). Find the eigenvalues $\lambda_1$, $\lambda_2$, $\lambda_3$ . (3%)

(2). Find the corresponding eigenvectors $E_1$, $E_2$, $E_3$ and matrix $E = \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}$. (6%)

(3). Find the orthogonal matrix $Q$ with respect to the eigenvectors and the diagonal matrix $D = Q^{-1}BQ$. (6%)