國立臺灣師範大學113學年度碩士班招生考試試題

科目：**機率與統計**　　　　適用系所：**數學系**

注意：1.本試題共1頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10分) How many ways are there to allocate 8 teachers to 3 schools if each school has at least 2 teachers.

2. (10分) An urn contains 3 red balls and 4 blue balls. A ball is drawn. If the ball is red, it is kept out of the urn and a second ball is drawn from the urn. If the ball is blue, then it is put back in the urn and a red ball is added to the urn. Then a second ball is drawn from the urn. What is the probability that the first drawn ball was blue, given that the second drawn ball is red?

3. (20分) The joint probability density of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{y}e^{-(y+x/y)} & x>0, y>0 \\ 0 & \text{otherwise.} \end{cases}$$

   Find $\mathrm{E}(X)$ and $\mathrm{Cov}(X,Y)$.

4. (20分) Let $X_1, \cdots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f(x;\theta) = \theta x\, e^{-\frac{1}{2}\theta x^2}, \quad x>0, \theta>0.$$

   (a) Find the maximum likelihood estimator of θ.

   (b) Find the method of moment estimator of θ.

5. (20分) Let X be a binomial distributed random variable with parameters 2 and θ. Suppose we want to test the null hypothesis $H_0 : \theta = 1/4$ against $H_1 : \theta = 3/4$ at the significant level at most $\alpha = 1/16$.

   (a) Find the rejection region R of the uniformly most powerful test at the significant level at most $\alpha = 1/16$.

   (b) Suppose we observe that $X = 2$. Find the p-value for this observation of the test.

6. (20分) For bivariate data on n cases $\{x_i, y_i\}$, consider the linear model with no intercept: $Y_i = \beta x_i + \varepsilon_i$, $i = 1, \cdots, n$ where ε_i are independent and identically $\mathcal{N}(0, \sigma^2)$ distribution with fixed but unknown variance $\sigma^2 > 0$.

   (a) Let $\hat{Y}_i = \hat{\beta} x_i$ be the fitted equation obtained by the least square method. Find the distribution of $\hat{\beta}$.

   (b) What is the distribution of the residuals sum of the squares, $\mathrm{SS}_{\mathrm{RSS}} = \sum_{i=1}^{n}(y_i - \hat{\beta}x_i)^2$, for the least square fit and what is the unbiased estimator of σ^2.

(試題結束)