113CC02

國立臺北科技大學 113 學年度碩士班招生考試

系所組別：2220 電子工程系碩士班乙組

第一節　機率　試題

注意事項：

1.本試題共 6 題，共 100 分。
2.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3.全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Two baseball teams, Texas Rangers and Baltimore Orioles, are going to play a best-of-five playoff series. The series ends as soon as one of the teams has won three games. Assume that each team is equally likely to win each game, there are no ties, and the outcomes of the games are independent.
   (a) Find the probability mass function $P_N(n)$ for the total number of games played in the series. (10%)
   (b) Find the probability mass function $P_W(w)$ for the number of wins by the Rangers in the series. (10%)

2. Let $X_1, X_2, \ldots, X_n$ be independent Gaussian random variables with $E\{X_k\} = 0$ and $Var\{X_k\} = a^k$ for $k = 1, 2, \ldots, n$, where a is a positive constant, and $E\{X_k\}$ and $Var\{X_k\}$ denote the mean and variance of X_k, respectively. Find the probability density function of $Y = \sum_{k=1}^{n} a^k X_k$. (10%)

3. The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.02. Given $T > 0.04$,
   (a) what is $E\{T|T > 0.04\}$, the conditional expected value of T? (10%)
   (b) what is $Var\{T|T > 0.04]$, the conditional variance of T? (10%)

4. Let U be a uniform random variable on (0, 1) and $X = -\ln(1-U)$, where $\ln(x)$ is the natural log function.
   (a) Find the cumulative distribution function of X. (10%)
   (b) Find the probability density function of X. (5%)
   (c) Find the expected value of X. (5%)

5. Random variables X and Y have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} cxy^2, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

   (a) What is the value of the constant c? (5%)
   (b) Find the probability of the event $\{Y < X^2\}$. (5%)
   (c) Find the probability of the event $\{\min(X, Y) \le 1/2\}$, where $\min(x, y)$ denotes the minimum of the two values x and y. (5%)

6. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with common probability density function

$$f_X(x) = \begin{cases} 2e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}.$$

   Let N be a geometric random variable with mean 3. Find the probability density function of $Y = X_1 + X_2 + \ldots + X_N$. (15%)