國立雲林科技大學 **113** 學年度
碩士班招生考試試題

系所：電子系
科目：工程數學(2)

1. (10%) About classifying differential equations.
   (a) (4%) How do we know if a differential equation is linear? And why do we need to confirm whether a differential equation is linear?
   (b) (6%) Identify the order, linearity, and homogeneity of each differential equation below,

$$5y''' + 3y' - 4\sin(y) + \cos(x) = 0$$
$$4y''' + 2y' = e^x y$$

2. (10%) Solving $(3xy - y^2)dx + x(x - y)dy = 0$

3. (15%) Solving $y'' - 2y' + y = \dfrac{e^x}{x^2}$

4. (15%) A system of differential equation is as follows,

$$\begin{cases} y_1' = +1y_1 + 2y_2 \\ y_2' = +3y_1 + 2y_2 \end{cases}$$

   (a) (5%) Find the solutions
   (b) (5%) Draw the phase portrait of this system
   (c) (5%) Identify the stability of this system and give your reasons.

5. (15%) Perform the indicated operation, give that

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 2 & 1 \end{bmatrix}$$

   (a) $(A+B)^T$ (b)$(2A-B)C$ (c) If $2X+4(A-B)=0$, Find X

6. (10%) Find the eigenvalues and eigenvectors of A.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

7. (15%) If $\vec{A} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{B} = \vec{i} + 3\vec{j} - \vec{k}$, Find (a) $\vec{A} \bullet \vec{B}$ (b) $\vec{A} \times \vec{B}$ (c) The projection of $\vec{A}$ on $\vec{B}$

8. (10%) Find the normal vector of the surface $z^2 = 2(x^2 + y^2)$ at the point P: (1, 0, 1)