國立中正大學
113 學年度碩士班招生考試
試題
[第2節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1) **(15 points)** Let $\Omega = \{1,2,3,4,5,6\}$ be the sample space. Define three events: $A = \{1,2\}$, $B = \{2,3\}$, and $C = \{4,5,6\}$. The probability measure is unknown, but it satisfies the three axioms of probability.
   a) **(5 points)** What is the probability of $A \cap C$?
   b) **(5 points)** What is the probability of $A \cup B \cup C$?
   c) **(5 points)** State a condition on the probability of either B or C that would allow them to be independent events.

2) **(20 points)** The probability density function (pdf) of a random variable X is shown in Fig. 1.

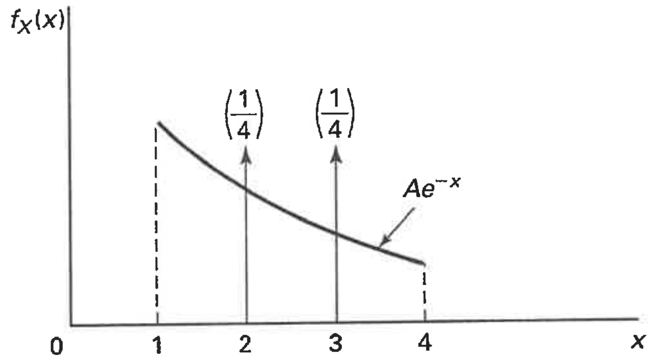

Fig. 1: pdf of X

   a) **(5 points)** Compute the value of A.
   b) **(10 points)** Find the cumulative distribution function (cdf) of X, that is, $F_X(x)$.
   c) **(5 points)** Compute the probability of the event $\{2 \le X < 3\}$.

3) **(25 points)** Consider a communication channel corrupted by noise. Let X be the value of the transmitted signal and Y the value of the received signal. Assume that the conditional density of Y given X is Gaussian, that is,

$$f_{Y|X}(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-x)^2}{2\sigma^2}\right)$$

and that X takes on only the values $+1$ and -1 equally likely.
   a) **(5 points)** Find the pdf of X, that is, $f_X(x)$.
   b) **(10 points)** Find the pdf of Y, that is, $f_Y(y)$.
   c) **(10 points)** What is the conditional density of X given Y, that is, $f_{X|Y}(x|y)$.

4) **(20 points)** Let X_1 and X_2 be independent and exponentially distributed random variables with pdf

$$f_{X_i}(x) = \frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right) u(x), \quad i = 1, 2,$$

where $\mu > 0$ and $u(x)$ is the unit step function. Define $Z \triangleq \max(X_1, X_2)$.

a) **(10 points)** Find the cdf of Z, that is, $F_Z(z)$.
b) **(10 points)** Find the pdf of Z, that is, $f_Z(z)$.

5) **(20 points)** In your physics courses, you have studied the concept of momentum $p = mv$ in the deterministic, that is, nonrandom sense. In reality, measurements of mass m and velocity v are never precise, thereby giving rise to an unavoidable uncertainty in these quantities. In this problem, we treat these quantities as random variables. So, consider a random variable mass M with given pdf $f_M(m)$ and a random variable velocity V with given pdf $f_V(v)$. We are also given the averages $\mu_M = E[M]$ and $\mu_V = E[V]$ (that would presumably correspond to our measurements in the physics course). Assume that M and V are independent and nonnegative random variables.

a) **(5 points)** Express the cdf of the momentum $P = MV$, that is, $F_P(p)$, in terms of the known pdf's $f_M(m)$ and $f_V(v)$.
b) **(10 points)** Find the pdf of $P = MV$, that is, $f_P(p)$, in terms of $f_M(m)$ and $f_V(v)$.
c) **(5 points)** Determine the expected value of the momentum $\mu_P = E[P]$ as a function of μ_M and μ_V.