國立中正大學
113 學年度碩士班招生考試
試 題
[第 2 節]

科目名稱	線性代數
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. Let $\boldsymbol{C}=\begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, $\boldsymbol{D}=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $\boldsymbol{E}=\begin{bmatrix} 0 & a \\ 1 & b \end{bmatrix}$. Answer the following questions with the appropriate matrix names (**C**, **D** or **E**). Note: No partial scores are given for each question.
   a. (5 pts.) Identify matrices that are row equivalent when a is 1 and b is 0.
   b. (5 pts.) Determine which matrix has $\{\boldsymbol{0}\}$ as the orthogonal complement of its row space when a is 0 and b is 1.
   c. (5 pts.) Which of these matrices satisfies the condition that the rank plus the nullity equals 2?
   d. (5 pts.) Which matrix is not full-rank when a and b are 1's?
   e. (5 pts.) Identify the matrix that contains row vectors that can span R^2 for all real numbers a and b, given that a is not equal to b.
   f. (5 pts.) Determine which matrix could be singular, given that the product of a and b is not equal to 0.

2. Determinant identity
   a. (10 pts.) For a given matrix $\boldsymbol{A}=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, identify all possible matrices **B**, where the last row of **B** is $\boldsymbol{0}$, such that the determinants of **AB** and **BA** are equal.
   b. (25 pts.) Prove that if **A** and **B** are matrices of sizes $m\times n$ and $n\times m$, then $\det(\boldsymbol{I}_m+\boldsymbol{AB})=\det(\boldsymbol{I}_n+\boldsymbol{BA})$.
   Hint: $\begin{bmatrix} I_n & -B \\ A & I_m \end{bmatrix}$.

3. Let $\boldsymbol{T}_1 : \mathbf{P}_1 \rightarrow \mathbf{P}_2$ be the linear transformation defined by $\boldsymbol{T}_1(\mathbf{p}(x)) = x\cdot\mathbf{p}(x)$ and let $\boldsymbol{T}_2 : \mathbf{P}_n \rightarrow \mathbf{P}_n$ be the linear operator defined by $\boldsymbol{T}_2(\mathrm{p}(x)) = \mathrm{p}(x+1)$, where $B = \{1, 2x\}$ and $B' = \{1, x, 2x^2\}$ are bases for $\mathbf{P}_1$ and $\mathbf{P}_2$, respectively. Every case requires detailed information.
   a. (10 pts.) Determine the coordinate vectors of $(x+1)_B$ and $(x^2+x)_{B'}$.
   b. (10 pts.) Represent the linear transformation $\boldsymbol{T}_1$ from $\mathbf{P}_1$ to $\mathbf{P}_2$ as the matrix $[\boldsymbol{T}_1]_{B\rightarrow B'}$.
   c. (10 pts.) Find the matrix representation $[\boldsymbol{T}_2]_{B'}$ of the linear transformation $\boldsymbol{T}_2$ in $\mathbf{P}_2$.
   d. (5 pts.) Show the matrix representation of the composition of two linear transformations $[\boldsymbol{T}_2\circ\boldsymbol{T}_1]_{B\rightarrow B'}$.